E. C. G. Sudarshan and the quantum mechanics of charged-particle beam optics

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E. C. G. Sudarshan had a multifaceted personality with immense contributions to diverse areas of physics. One of his contributions not widely known is to do with the emerging field of 'quantum mechanics of charged-particle beam optics'. The present note elaborates on the origins and the development of this emerging field.

E. C. G. Sudarshan had a multifaceted personality with immense contributions to diverse areas of physics[1,2]. His contributions to particle physics and quantum theory are part and parcel of standard textbooks in physics. His innovative discoveries in the field of quantum optics (such as the optical equivalence theorem, which provides the foundation on which the investigations of the manifestly quantum or non-classical character of the electromagnetic field are based) are well known. One of his contributions not as widely known is to do with the emerging field of 'quantum mechanics of charged-particle beam optics'. The present note elaborates on the origins and development of this emerging field.

In a series of papers starting in 1983, Mukunda *et al.*[3] had established, using a group theoretical approach, Fourier optics for the Maxwell field which generalizes the paraxial scalar wave optics to paraxial Maxwell wave optics, consistently taking into account polarization of light. When Sudarshan became the Director of the Institute of Mathematical Sciences (IMSc), Chennai (1985–1990), Jagannathan, then a junior faculty member of the Institute, started to assist him in teaching a course on quantum mechanics to Ph D students at IMSc. Jagannathan used to take down notes of Sudarshan's lectures and whenever the latter was absent for a few days, Jagannathan filled the gap by his own lectures on the topic starting from where Sudarshan had left it. When Sudarshan returned, he continued the lectures from where Jagannathan had left. Jagannathan used to write down the notes of his lectures on the left side of a notebook in which Sudarshan wrote down the notes of his lectures on the right side. During the course of this collaboration, Sudarshan suggested to Jagannathan to study the group theoretical aspects of electron optics based on the Dirac equation, with the aim to extend the traditional nonrelativistic scalar wave theory of electron optics to a Dirac spinor wave theory taking into account the spin of the electron analogous to the way they had extended the scalar light optics to the Maxwell vector optics taking into account the polarization of light. Jagannathan had the experience of studying the group theoretical aspects and physical applications of generalized Clifford algebras, which are generalizations of the algebra of Dirac matrices, for his Ph D thesis. A literature search led to a surprise: electron optics, or in general, charged-particle optics, used in beam devices from electron microscopes to large particle accelerators, was based essentially on classical mechanics and classical electrodynamics. Of course, in electron microscopy, image formation was understood based on the nonrelativistic Schrödinger equation, and electron spin and the Dirac equation were not mentioned at all. In accelerator physics, topics like synchrotron radiation were treated using quantum field theoretic techniques, but the design and operation of the accelerator beam elements were based only on classical mechanics and classical electrodynamics. So, a study programme was started from scratch to understand the quantum mechanics of electron optics based on the Dirac equation. The very first result from this study was the derivation of the focusing action of a round magnetic lens, the central part of any electron microscope, *ab initio* using the Dirac equation by Jagannathan *et al.*[4]. Sudarshan encouraged Jagannathan to continue the study further in this direction and a subsequent paper by the latter still serves as the blueprint for the quantum mechanics of charged-particle beam optics[5].

When I became a Ph D student of Jagannathan in 1992, he suggested that I look into extending their work on the quantum theory of electron optics, based on paraxial approximation, to aberrating systems using the Heisenberg and interaction pictures, which should enable going beyond the paraxial approximation. Closely following Jagannathan's 1990 blueprint[5], the quantum theory of electron optical aberrations was developed using the Klein–Gordon and Dirac equations respectively[6,7], with applications in both electron microscopy and accelerator optics. It was further shown that the quantum framework delivers a unified theory of the beam optics and spin dynamics of a Dirac particle, taking fully into account the anomalous magnetic moment. The paraxial limit of this approach readily leads to the beam-optical version of the Thomas–Bargmann–Michel–Telegdi (Thomas–BMT) equation[8]. These works eventually led to the first doctoral thesis on the subject of quantum theory of charged-particle beam optics under the supervision of Jagannathan[9].

The understanding of the quantum theory of charged-particle beam optics initiated by Sudarshan and co-workers, and carried forward by Jagannathan had a decisive influence on the prescriptions of light beam optics using 'quantum methodologies'. The Helmholtz equation governing scalar optics is algebraically similar to the Klein–Gordon equation[10–12]. The beam optical Hamiltonian derived from an exact matrix representation of Maxwell's equations also has the algebraic structure of the Dirac equation[13]. This beam optical Hamiltonian governs the vector optics. The great similarities in the underlying mathematical structures of the two systems ensure that we can use the powerful machinery of relativistic quantum mechanics. The Foldy–Wouthuysen transformation technique is crucial in both the cases. The beam-optical Hamiltonians thus obtained naturally have wavelength-dependent components in both the scalar[10–12] and vector[14–16] cases. In the limit of low wavelength, non-traditional prescriptions of Helmholtz optics and Maxwell optics lead to the traditional descriptions of light-beam optics. This corresponds to taking the classical limit of quantum prescriptions. Furthermore, in the vector case, it leads to a unified prescription of beam optics and light polarization[14–16]. The aforementioned formalisms extend Hamilton's optical–mechanical analogy into the wavelength-dependent regime[17]. Some of these historical aspects were touched upon during the United Nations-designated '2015 International Year of Light and Light-based Technologies'[18]. The non-traditional prescriptions of Helmholtz optics and matrix formulation of Maxwell optics using quantum methodologies are among the 30 research summaries selected for 'Optics in 2016' by the Optical Society of America, highlighting the most exciting peer-reviewed optics research to have emerged during that year[19].

This is a brief account of how Sudarshan's suggestion led to the emerging field of 'quantum mechanics of charged-particle beam optics', and how it further influenced new formalisms of light-beam optics. A book by Jagannathan and the present author on the quantum mechanics of charged-particle beam optics is under way[20].

1. Yajnik, U. A., *Curr. Sci*., 2018, **114**(12), 2565–2569.
2. Mukunda, N., *J. Phys.: Conf. Ser*., 2009, **196**, 012009.
3. Mukunda, N., Simon, R. and Sudarshan, E. C. G., *J. Opt. Soc. Am. A*, 1985, **2**(3), 416–426.
4. Jagannathan, R., Simon, R., Sudarshan, E. C. G. and Mukunda, N., *Phys. Lett. A*, 1989, **134**, 457–464.
5. Jagannathan, R., *Phys. Rev. A*, 1990, **42**(11), 6674–6689.
6. Khan, S. A. and Jagannathan, R., *Phys. Rev. E*, 1995, **51**(3), 2510–2515.
7. Jagannathan, R. and Khan, S. A., *Adv. Imag. Electron Phys*., 1996, **97**, 257–358.
8. Conte, M., Jagannathan, R., Khan, S. A. and Pusterla, M., *Part. Accel*., 1996, **56**, 99–126.
9. Khan, S. A., *Quantum theory of charged-particle beam optics*. Ph D thesis, University of Madras, Chennai, 1997; http://www.imsc.res.in/xmlui/handle/123456789/75?show=full
10. Khan, S. A., Jagannathan, R. and Simon, R., Foldy–Wouthuysen transformation and a quasiparaxial approximation scheme for the scalar wave theory of light beams, 2002; arXiv:physics/0209082 [physics.optics].
11. Khan, S. A., *Int. J. Theor. Phys*., 2005, **44**(1), 95–125.
12. Khan, S. A., *Optik – Int. J. Light Electron Opt*., 2006, **117**(10), 481–488; 2016, **127**(20), 9798–9809; 2018, **153C**, 164–181.
13. Khan, S. A., *Phys. Scr*., 2005, **71**(5), 440–442.
14. Khan, S. A., *Optik – Int. J. Light Electron Opt*., 2010, **121**(5), 408–416; 2014, **125**(3), 968–978; 2017, **131**, 350–363; 2017, **131**, 733–748.
15. Khan, S. A., *Adv. Imag. Electron Phys*., 2008, **152**, 49–78; 2017, **201**, 57–135.
16. Khan, S. A., *J. Mod. Opt*., 2016, **63**(17), 1652–1660.
17. Khan, S. A., *Optik – Int. J. Light Electron Opt*., 2017, **130C**, 714–722.
18. Khan, S. A., *Curr. Sci*., 2014, **106**(4), 501; 2016, **111**(4), 627–631.
19. Khan, S. A., *Opt. Photon. News*, 2016, **27**(12), 47; http://www.osa-opn.org/home/articles/volume_27/december_2016/features/optics_in_2016/
20. Jagannathan, R. and Khan, S. A., *Quantum Mechanics of Charged Particle Beam Optics*, CRC Press, Taylor & Francis, London, UK, 2019; http://isbn.nu/9781138035928/

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