Identifying the design skies for Indian tropical climatic conditions

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The current Indian design sky model is inadequate for daylighting design and calculation of annual daylight energy savings in buildings due to which designers are compelled to resort to other sky luminance prediction models like Perez and International Commission on Illumination (CIE) sky models. Perez and CIE sky models are derived from the sky scan data of mostly sub-tropical and temperate regions where the climatic conditions are different from the tropical climatic conditions. The present study attempts to check and compare the adequacy of Perez and CIE sky models for prediction of sky luminance in Indian tropical climatic conditions by comparing model-predicted values with the measured values from the past studies. Further, a framework has been proposed for the identification of CIE design skies for passive window design in the locations where measured sky luminance data is not available. CIE design skies have been recommended for the major climatic zones of the tropical climate of India using the proposed framework.

Keywords: Design sky, passive window design, daylighting, Indian tropical region.

INDIAN Standard Code IS 2440 (ref. 1) recommends a design sky model for passive window design in India. Accordingly, the sky luminance at an altitude angle of θ is given as

$$L_{v} = L_{z} \operatorname{cosec} \theta, \tag{1}$$

where L_{ν} is the luminance of the sky element at the altitude angle of θ and L_z is the sky luminance at zenith. The formula is applicable for sky elements lying between 15° and 90° sky altitude with uniform brightness below 15° sky altitude. Uniform luminance is assumed along the azimuthal direction for a given sky altitude angle. The design sky given in the code has been derived based on the measurements taken at CBRI (Central Building Research Institute, Roorkee, India) at 15° solar altitude in 1975 (refs 1, 2). Basic hypothesis for above design sky is: 15° altitude represents the time corresponding to that at one hour after sunrise (7 a.m. or so) or one hour before sunset (5 p.m. or so), covering complete working hours. The daylight availability would be minimum during working hours at that time and represents the lower limit of sky luminance for design purpose. Passive window design with a low solar altitude design sky ensures daylight availability through fenestration throughout the working hours and is, therefore, a reasonable assumption. It was further assumed that fenestration design for daylight would prefer window on north wall to exclude direct sun light so as to avoid glare and heat admission. Thus diffused sky on north-east or north-west quadrant of the sky hemisphere opposite to the sun's position was considered in the morning and afternoon respectively. Hence the assumption of azimuthal uniformity was adopted. But assuming azimuthal uniformity may lead to an inefficient design, which is explained as follows.

According to the measured sky luminance data of other tropical countries and established sky luminance prediction models^{3,4}, the sky luminance varies as the distance from the sun changes (both in azimuth and altitude). A good passive window design requires precise prediction of sky luminance in every direction of the sky dome. Apart from this, Indian design sky does not take into consideration the change in sky conditions with the change in climatic zones. It is also inadequate for calculation of annual energy savings due to daylight as it gives a single sky luminance distribution for a low solar altitude angle and leads to underestimation of the daylight availability. The shortcomings of the current Indian design sky model as described above are azimuthal uniformity, climate invariant and inadequacy for calculation of annual energy savings due to daylight. These shortcomings compel the designers to employ other sky luminance prediction models for window design purposes and annual building energy load calculations. Perez' and International Commission on Illumination (CIE) sky models⁶ are widely accepted models for the prediction of sky luminance distribution. Daylight simulation softwares used by architects like Gendaylit Sky Generator in Radiance⁷, Daysim⁸ and Daylight Visualizer⁹ use Perez and CIE sky models for prediction of sky luminance distribution.

The coefficients used for the prediction of sky luminance in Perez and CIE sky models are derived from the regression analysis of the measured data from countries mostly falling in subtropical and temperate regions. The climatic conditions of tropical regions differ considerably

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from that prevalent in sub-tropical and temperate regions. It is still a matter of question if the sky luminance prediction model derived from the sky scan data of the sub-tropical and temperate climatic region predicts sky luminance correctly for Indian tropical climatic regions as well. In the present study, an attempt has been made to check the adequacy of Perez and CIE sky luminance prediction models for Indian tropical climatic conditions by comparing the model predicted values with the measured values available from past studies. The unavailability of measured luminance data limits the use of the CIE sky model for passive window design in many countries. Therefore, a framework has been developed to propose CIE design skies for passive window design using the irradiance data for the locations where measured luminance data is not available. A set of design skies have been proposed for the major climatic zones of the Indian tropical climate using the proposed framework.

Literature review

A few authors have attempted to validate the accuracy of Perez and CIE sky models for sky luminance prediction in tropical regions and have also identified suitable standard CIE sky types for countries in the tropical climatic region. Tregenza¹⁰ measured the sky luminance distribution at the stations in Singapore, Japan, Garston (UK) and Sheffield (UK) and compared it with the modelled sky luminance of the 15 standard skies proposed by CIE⁶ and Perez sky model⁵ using root mean square error method to check the adequacy of the proposed sky models in representing the actual sky conditions occurring under the climatic condition of these cities. Mukherjee¹¹ attempted to identify a suitable sky type for Delhi (India) by comparing the ratio of zenith luminance to horizontal diffuse illuminance for the CIE standard skies with the measured values for Delhi. Diffuse horizontal illuminance was calculated by an empirical model given in the CBRI report. In the study, CIE sky types 8 and 6 (partly cloudy sky) were identified as suitable skies for summer and winter seasons respectively. Mukherjee and Roy¹² again attempted to find out the suitable sky type for Roorkee in three different seasons through comparison of the spatial illuminance values obtained on a working plane using the CIE standard skies and the measured sky luminance values. The results revealed that CIE clear sky of type 15 (white-blue turbid sky) is suitable for luminance predictions in summer and equinox seasons and CIE clear sky of type 11 (white-blue sky) for the winter season. Li et al.¹³ measured the sky luminance for Hong Kong sky and compared the sky luminance distribution with modelled sky luminance distribution for the 15 CIE standard skies by the root mean square error method to arrive at a subset of three sky types which were sufficient to describe the sky occurrence in subtropical Hong Kong. Mettanant *et al.*¹⁴ tried to develop a new method to model sky luminance distribution of Thailand using the reduced set from the CIE standard skies and optimized limit values of the Perez's clearness and sky brightness parameters for accurate sky classification. The authors used the stochastic search technique of evolutionary computation for obtaining the optimized limit values.

It can be seen from the literature review that the researchers in the past have attempted to propose methodologies and have identified suitable CIE sky types for some tropical countries, but a similar type of comprehensive study for Indian tropical climate has not been done. Also, there seems to be no readily available study to propose design skies for the passive design of windows for daylighting in the tropical climatic zone. Indian tropical conditions are different from other tropical countries because of modifications due to the massive mountainous range of the Himalayas in the north and oceans in the south. The following section gives an overview of the Indian tropical climate and how it is different from the climate of other countries in the same latitude.

Tropical zones of India

Most parts of India fall under tropical region as it spans from 8° lat. in Kanyakumari to 34° lat. in Leh. There are massive Himalayan mountain ranges in the north and oceans such as the Indian Ocean, Arabian Sea and Bay of Bengal surrounding the southern part of the country. Besides these, other mountainous ranges of lesser altitudes are also present, e.g. the Eastern Ghats, Western Ghats and Nilgiris in the southern peninsula and Satpura, Vindhyachal and Aravali in the central part of the country. The topographical features induce climatic changes that make Indian climatic conditions different from the other countries in the same tropical region. One major difference is the existence of a specific climate zone as a composite monsoon climate which is not prevalent in most of the other tropical locations. Monsoon is triggered from the oceans near tropic of cancer during summer, whereby moisture-bearing winds travel towards the north and cause precipitation when they come in contact with the mountainous range of Himalayas. This phenomenon modifies the climatic condition in the major part of North and Central India. Besides, Northern polar winds get blocked by the same mountainous ranges causing moderation of the winter conditions. However, the countries falling under the same latitude range experience different climatic conditions. For example, the latitude of Arizona and part of Northern India is almost the same $(30 \pm 3^{\circ}N)$. However, while part of Arizona is a desert, similar latitude locations in India experience composite climate.

The National Building Code^{15} classifies the country into five climatic zones, namely (1) hot and dry, (2) warm

Climatic zone	Mean monthly maximum temperature (°C)	Mean monthly relative humidity percentage		
Hot and dry	Above 30	Below 55		
Warm and humid	Above 30	Above 55		
	Above 25	Above 75		
Temperate	Between 25–30	Below 75		
Cold	Below 25	All values		

 Table 1.
 Classification of climatic zones (adapted from BIS 2005)

Table 2. Measured luminance data

	Season/		
Place	date of measurement	Solar altitude	Source
Roorkee	March equinox/21 March	59.74°	Mukherjee and Roy ^{2,3}
Roorkee	Summer solstice/21 June	82.57°	Mukherjee and Roy ^{2,3}
Roorkee	Winter solstice (25 December)	38.33°	Mukherjee and Roy ^{2,3}
Delhi	Winter	20°	Sastri and Manmohan ⁴
Delhi	Winter	40°	Sastri and Manmohan ⁴
Mumbai	Winter/11 December 1963	30°	Narsimhan and Saxena ⁵
Nagpur	Winter/13 January 1964	45°	Narsimhan and Saxena ⁵

and humid, (3) temperate, (4) cold and (5) composite. The criteria for the classification of these climatic zones are the mean monthly maximum temperature and mean monthly percentage relative humidity. Table 1 gives the range of temperature and relative humidity for the classification of these climatic zones. Each zone experiences particular weather conditions for more than six months whereas there may be a change in the weather for a few months. Hot and dry climatic zones experience high maximum temperature, high diurnal ranges and low humidity for most part of the year. Warm and humid zones experience somewhat lower maximum temperatures, lower diurnal ranges with high humidity. Temperate climatic zones experience moderate temperatures with moderate humidity, whereas cold climatic zones have low temperatures during most part of the year. The composite climatic zone does not have any weather conditions continuously for more than six months.

Methodology

For checking the adequacy of the Perez and CIE sky model in predicting sky luminance distribution under Indian tropical conditions, the model predicted values were checked against measured values taken from past studies for some major cities in India. A summary of the measured sky luminance data from past studies and the Perez and CIE sky model is given in the following sections.

Measured sky luminance data

Data for four cities such as Roorkee, Delhi, Nagpur and Mumbai in different seasons were taken from the literature. Sky luminance data for Roorkee were measured at CBRI as a part of the International Daylight Measurement Program¹². The measurements were taken for the whole sky vault in three minutes using a sky scanner from M/s EKO Japan, having a field view of 11° at 12:00 noon for the three seasons of the summer solstice, winter solstice and March equinox. The data were recorded at 12 intervals of altitude starting from 6° above the horizon and every 30 azimuthal intervals for a total of 145 points on the entire sky vault¹². The measured luminance values were plotted in the form of iso-luminance contours¹². Sky luminance data for Delhi were measured¹⁶ using a sky scanning photometer and were plotted in the form of isoluminance contours for 20° and 40° solar altitude for the winter months of 1969-70. Luminance data for Mumbai and Nagpur were measured by Narsimhan and Saxena² at solar altitudes of 30° and 45° respectively and the data were plotted in the form of iso-luminance contours. The above-mentioned set of measured data for four locations in different seasons were used to check the adequacy of Perez and CIE sky models for sky luminance prediction. Table 2 lists all the measured luminance data along with their source, date/season of measurement, place of measurement and solar altitude at the time of measurement. The luminance values plotted in the form of contour lines were linearly interpolated to get the luminance value between two successive contours lines. Luminance values were divided by the luminance at zenith in the contours to obtain relative luminance values. Table 3 shows the relative luminance values obtained for Roorkee city in all the three seasons.

Sky models

The CIE sky model is the current standard for sky luminance prediction as adopted by the International

Table 3. Measured relative sky luminance values for Roorkee												
Azimuth	0	30	60	90	120	150	180	210	240	270	300	330
Altitude (Roo	orkee summe	r solstice)										
10	0.20	0.21	0.15	0.22	0.25	0.24	0.24	0.21	0.20	0.15	0.14	0.17
20	0.22	0.23	0.24	0.27	0.29	0.28	0.27	0.26	0.25	0.23	0.21	0.21
30	0.25	0.25	0.26	0.30	0.32	0.32	0.31	0.29	0.27	0.26	0.24	0.23
40	0.27	0.27	0.28	0.35	0.38	0.39	0.37	0.33	0.31	0.28	0.26	0.26
50	0.29	0.30	0.32	0.42	0.51	0.53	0.53	0.43	0.36	0.31	0.30	0.28
60	0.34	0.36	0.42	0.57	0.76	0.95	0.93	0.60	0.47	0.38	0.34	0.33
70	0.44	0.51	0.60	0.88	1.25	1.20	1.40	0.83	0.65	0.55	0.45	0.44
80	0.66	0.75	0.86	1.15	1.52	1.20	1.40	0.91	0.86	0.76	0.66	0.66
90	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Altitude (Roo	orkee winter s	solstice)										
10	0.50	0.50	0.50	1.00	1.56	5.00	6.50	2.25	1.29	0.75	0.75	0.75
20	0.75	0.75	0.75	1.00	2.00	5.00	6.50	2.71	1.43	1.00	0.75	0.75
30	0.75	0.75	0.75	1.00	2.00	5.00	9.06	3.07	1.47	1.00	0.75	0.75
40	0.75	0.75	0.75	1.00	1.96	5.00	10.00	3.06	1.42	1.00	0.75	0.75
50	0.75	0.75	0.75	1.00	1.77	4.58	7.08	2.69	1.37	1.00	0.75	0.75
60	0.75	0.75	0.77	1.00	1.58	3.07	3.54	2.32	1.32	1.00	0.75	0.75
70	0.75	0.75	0.85	1.00	1.38	2.08	1.75	1.96	1.27	1.00	0.75	0.75
80	0.81	0.86	0.92	1.00	1.19	1.21	1.16	1.57	1.15	1.00	0.84	0.82
90	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Altitude (Roo	orkee march e	equinox)										
10	0.50	0.50	0.40	0.67	0.86	1.24	1.35	1.00	0.83	0.54	0.52	0.50
20	0.50	0.50	0.52	0.70	1.03	1.46	1.58	1.13	0.84	0.67	0.55	0.50
30	0.50	0.50	0.55	0.74	1.12	1.76	2.20	1.28	0.89	0.67	0.57	0.50
40	0.50	0.50	0.58	0.78	1.21	2.78	3.13	1.50	0.94	0.67	0.60	0.50
50	0.50	0.51	0.62	0.83	1.36	4.38	6.11	1.67	0.99	0.72	0.62	0.50
60	0.50	0.58	0.65	0.87	1.50	4.70	5.83	1.67	1.00	0.79	0.65	0.55
70	0.61	0.65	0.73	0.91	1.67	3.67	3.54	1.67	1.00	0.86	0.71	0.64
80	0.78	0.81	0.87	0.96	1.33	1.50	1.56	1.33	1.00	0.93	0.83	0.78
90	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Commission on illumination¹⁷. A set of fifteen standard skies were proposed by Darula and Kittler⁶ derived from the sky scan measurement data recorded in Tokyo, Berkeley and Sydney as a part of the International Daylight Measurement program (IDMP) which was later adopted by CIE as a standard. CIE sky model requires the measured value of zenith luminance and horizontal diffuse illuminance for the selection of sky type from the representative set of possible sky luminance distributions. Sky luminance or illuminance is not measured at the Indian Meteorological stations and thus limits the use of the CIE sky model in countries like India. CIE model defines a total of 15 possible sky luminance distributions. The table given in the standard¹⁷ lists six groups of k and l values for the gradation function and six groups of m, n and pvalues for the indicatrix function. A total of 15 combinations of these coefficients define the change in relative luminance with the zenith angle and angular distance of the sky element from the sun. For future reference, the table has been reproduced here as Table 4. The table defines the sky conditions corresponding to each sky type and the corresponding values of the parameters. For the identification of CIE sky type from the set of skies given in the standard, the relative luminance from the measured luminance data was compared against the relative luminance predicted by 15 CIE sky types for each of the cities and seasons. The CIE sky type having the least error for a particular city and season was chosen as the CIE sky type for that city and season. These chosen CIE sky types for different cities were then used to assess the accuracy of the CIE sky model in the comparative study.

Perez sky model was developed based on the sky scan data measured from ten American and three European cities⁵. The model relates the sky luminance distribution to the insolation conditions through coefficients which define the sky conditions for a particular location, day and time of the year. Perez model is more versatile in the sense that the parameters used for luminance prediction are derived from irradiance measurements at the place under consideration, the data for which is easily available at most of the meteorological stations. For the calculation of Perez coefficients, solar radiation data for all the cities except Roorkee were taken from the solar radiation handbook¹⁸. In this handbook¹⁸, radiation data is given for the representative year or typical meteorological year of the respective cities which was developed by Sandia National Laboratories, USA¹⁹. The typical meteorological year is a dataset in which hourly values of solar radiation

	I able 4. International Commission on Illumination (CIE) sky types (adapted from Darula and Kittler")												
Туре	Gradation	Indicatrix	k	l	т	n	р	Description of luminance distribution					
1	Ι	1	4	-0.7	0	-1	0	CIE Standard Overcast Sky, alternative form steep luminance gradation towards zenith, azimuthal uniformity					
2	Ι	2	4	-0.7	2	-1.5	0.15	Overcast, with steep luminance gradation and slight brightening towards the sun					
3	II	1	1.1	-0.8	0	-1	0	Overcast, moderately graded with azimuthal uniformity					
4	II	2	1.1	-0.8	2	-1.5	0.15	Overcast, moderately graded and slight brightening towards the sun					
5	III	1	0	-1	0	-1	0	Sky of uniform luminance					
6	III	2	0	-1	2	-1.5	0.15	Partly cloudy sky, no gradation towards zenith, slight brightening towards the sun					
7	III	3	0	-1	5	-2.5	0.3	Partly cloudy sky, no gradation towards zenith, brighter circumsolar region					
8	III	4	0	-1	10	-3	0.45	Partly cloudy sky, no gradation towards zenith, distinct solar corona					
9	IV	2	-1	-0.55	2	-1.5	0.15	Partly cloudy, with the obscured sun					
10	IV	3	-1	-0.55	5	-2.5	0.3	Partly cloudy, with brighter circumsolar region					
11	IV	4	-1	-0.55	10	-3	0.45	White-blue sky with distinct solar corona					
12	V	4	-1	-0.32	10	-3	0.45	CIE Standard Clear Sky, low illuminance turbidity					
13	V	5	-1	-0.32	16	-3	0.3	CIE Standard Clear Sky, polluted atmosphere					
14	VI	5	-1	-0.15	16	-3	0.3	Cloudless turbid sky with broad solar corona					
15	VI	6	-1	-0.15	24	-2.8	0.15	White-blue turbid sky with broad solar corona					

Table 5.	Parameters and their respective weightage
factors u	sed for arriving typical meteorological year in
solar ra	diation handbook (adapted from IMD 2009)

Parameter	Weightage factor
Global solar irradiance	7/23
Diffuse solar irradiance	6/23
Mean air temperature	2/23
Maximum air temperature	1/23
Minimum air temperature	1/23
Mean dew point temperature	2/23
Maximum dew point temperature	123
Minimum dew point temperature	1/23
Mean wind speed	2/23

and meteorological elements are selected in the form of months from individual years and later concatenated to form a complete year. The data collected in this form gives a standard data set that has frequency distribution close to the long term distribution. The sandia method uses nine parameters for selecting a representative month and each parameter is assigned a specific weightage. Table 5 gives the parameters used to arrive at the typical meteorological year in the solar radiation handbook. As can be seen, a large weightage has been assigned to global and diffuse irradiation. The data obtained from the typical meteorological year can be assumed to be representative of the long term solar radiation data for the locations. The data set for Roorkee was obtained by averaging 15 years (2000-2014) of irradiation data obtained from the National solar radiation database²⁰ as it was not available in the solar radiation handbook.

Prediction and comparison of sky luminance data

For checking and comparing the accuracy of the two sky models, the whole sky dome was divided into sky patches at 10° intervals of altitude starting from 10° above the horizon to 80° sky altitude at every 30° azimuthal interval resulting in a total of 96 sky patches on the entire sky vault. Relative sky luminance was predicted using the Perez sky model and the chosen CIE sky type of the CIE sky model at the centre of these 96 sky patches. Relative luminance at the centre of the sky patches was also obtained from the iso-luminance contours of the measured data from the past studies as shown in Table 3.

Root mean square error and mean bias error metrics were used to test how well the predicted values matched with the measured values. These metrics were used for their ability to evaluate the nature of differences between predicted values of models against the observed values as recommended by researchers²¹. Mean bias error was used to get an idea about the average bias prediction of the luminance values. The formula used for calculating the root mean square error and mean bias error values is as given below.

Mean bias error

$$MBE = \frac{1}{n} \sum_{i=1}^{n} \frac{x_i - \overline{x}}{\overline{x}}.$$
 (2)

Root mean square error

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left(\frac{x_i - \overline{x}}{\overline{x}}\right)^2}.$$
 (3)

City/season	CIE sky type	RMSE (%)	MBE (%)	Standard deviation	Percentage error	Correlation coefficient
CIE						
Roorkee/winter solstice	11	19	2.60	0.19	11.75	0.97
Roorkee/summer solstice	7	18.69	-1.70	0.19	14.48	0.9
Roorkee/March equinox	11	20.30	-4.20	0.2	14.25	0.908
Perez						
Roorkee/winter solstice	_	20.89	7.65	0.19	12.87	0.97
Roorkee/summer solstice	_	25.75	-21.56	0.14	22.93	0.91
Roorkee/March equinox	-	22.41	-5.80	0.22	16.83	0.91

Table 6. Error metrics for best fit CIE sky and Perez model predicted values for Roorkee city

Table 7. Error metrics for best fit CIE sky and Perez model predicted values for Delhi city

City/season	CIE sky type	RMSE (%)	MBE (%)	Standard deviation	Percentage error	Correlation coefficient
CIE						
Delhi 20° solar altitude/winter	10	24.48	-6.45	0.24	16.67	0.8
Delhi 40° solar altitude/winter	11	27.46	-3.89	0.25	18.87	0.85
Perez						
Delhi 20° solar altitude/winter	_	29.41	-1.67	0.29	20.27	0.77
Delhi 40° solar altitude/winter	-	26.95	2.10	0.25	18.63	0.84

Here the \overline{x} represents the measured values obtained from the iso-luminance contours and x_i represents the predicted luminance values by the Perez and CIE sky model.

Apart from the above mentioned metrics, percentage error, standard deviation and coefficient of correlation were also computed between the predicted and measured values.

Results and discussion

The error metrics between the model predicted values and measured data for all three seasons of Roorkee is shown in Table 6. The measured sky luminance values at 10° sky altitude for winter solstice were not considered in the comparative study as the inclusion resulted in an increase of 20% in the root mean square error value. The measurement at low sky altitude is prone to error as a large part of the path of light traverses through the atmosphere close to the earth. Due to the presence of suspended particles and moisture in the atmosphere close to the earth, scattering of the light rays occurs, which affects the luminance measurement at the stations and may lead to an erroneous measured luminance value. As can be seen from the table, the CIE model fits the sky luminance distribution better than the Perez model at 12 noon for Roorkee city in all the three seasons. The CIE model has the least error during the summer solstice season whereas the Perez model has the least error during the winter solstice season. The CIE sky type 11 was chosen as the sky type for the winter solstice and March equinox seasons and CIE sky type 7 for the summer solstice season for prediction using the CIE sky model. Mean bias error shows that both Perez and CIE sky model overestimate the relative luminance in the winter season and underestimate it in the other two seasons. The root mean square error and percentage error in the Perez model predicted values show an increase with the rise in solar altitude, whereas there is no specific pattern for the CIE model predicted values. Standard deviation is around 0.2 for CIE model predicted value while it varies from 0.14 to 0.22 for Perez model predicted values. Coefficient of correlation is above 0.9 for both the models.

The error metrics between model predicted values and measured data for Delhi at different solar altitudes are shown in Table 7. It can be seen from the table that the CIE model has a lower root mean square error at 20° solar altitude but a slightly higher root mean square error at 40° solar altitude compared to the Perez model predicted values. The error is higher at lower solar altitude for the Perez sky model whereas it is lower at lower solar altitude for the CIE sky model. The CIE sky type changes with the variation in solar altitude. The partly cloudy CIE skies of type 10 and 11 were found to be suitable sky types for prediction of sky luminance distribution at 20° and 40° solar altitudes respectively using the CIE sky

Table 6. El	Tor metrics for desi	. III CIE SKY a	nu relez moue	i predicted valu	les for Mullioar	and Nagpur City
City/season	CIE sky type	RMSE (%)	MBE (%)	Standard deviation	Percentage error	Correlation coefficient
CIE						
Mumbai/winte	er 10	14.60	-5.60	0.2	11.34	0.95
Nagpur/winter	r 10	11.90	-7.90	0.16	9.47	0.97
Perez						
Mumbai/winte	er –	19.63	1.50	0.13	12.53	0.93
Nagpur/winter	r –	17.13	-4.70	0.09	14.04	0.94

 Table 8.
 Error metrics for best fit CIE sky and Perez model predicted values for Mumbai and Nagpur city

model. Standard deviation is around 0.25 for CIE model whereas it goes up to 0.29 for the Perez model predicted values. Coefficient of correlation is around 0.8 for both the models with lower correlation coefficient of 0.77 at 20° solar altitude in Delhi climate.

The relative luminance from the measured values for Nagpur and Mumbai city obtained from Narasimhan and Saxena's study² were compared against the Perez and CIE model predicted values for the same locations. The results are shown in Table 8. CIE model predicts the sky luminance distribution better than the Perez model for both Mumbai and Nagpur city. The CIE sky type 10 was found to be the suitable sky type for both Mumbai and Nagpur region. CIE sky model underestimates the luminance values for Mumbai, whereas there is no specific pattern shown by the Perez sky model. The root mean square error for both CIE and Perez sky model falls below 30% for all the cities and seasons. Standard deviation is lower in the Perez model predicted values, which is unlike the measured values for other two cities. Coefficient of correlation is higher for CIE model predicted values.

From the results of the comparison study, it can be concluded that the CIE sky model predicts sky luminance distribution better than the Perez sky model if one knows the correct sky type. Identification of the correct sky type requires measured luminance data which is not easily available in India. A wrong sky type may result in an error in predicted sky luminance values and lead to inefficient design. In the absence of any luminance data, the suitable CIE sky types identified in the present study can be used for prediction of sky luminance for places in the vicinity of the current cities and with similar climatic conditions. In the absence of sky luminance data and knowledge of the suitable CIE sky type, one can still resort to the Perez sky model as it still predicts the sky luminance distribution within reasonable accuracy.

As the CIE sky model is the current standard of sky luminance prediction and is used by many softwares for daylighting design, a methodology is needed to identify a suitable sky type for the daylighting design of windows in the location where measured luminance data are not available. The following section presents a framework to identify a suitable design CIE sky type for passive window design in such locations.

Framework for identifying CIE design sky for various locations

For the passive design of windows for daylighting, a low solar altitude sky luminance distribution is used. Choosing sky luminance distribution at low solar altitude ensures the presence of sufficient daylight throughout the working hours and hence is a suitable assumption for selection of design sky. Daylight illumination levels are quite low to be used for task illumination in the first hour after sunrise. After the first hour, sufficient daylight is available to be considered for utilization in task illumination inside the building; therefore 15° solar altitude luminance distribution is considered for the design sky. As the sky luminance distribution also changes with weather conditions during the year, window design based on a sky type having minimum illumination among the twelve months will ensure sufficient daylight during the whole year. A CIE sky type satisfying the abovementioned conditions of a design sky needs to be identified for locations without measured luminance data. Perez sky model can be utilized for the purpose as explained in the following paragraph.

It can be seen from Tables 5 to 7 that the root mean square deviation of the CIE and Perez model predicted values from the measured values are close to each other and the difference between them is below 7%. The root mean square deviation and mean bias deviation of the Perez and CIE model predicted values from the measured data are comparable and close to each other. It can be deduced from this observation that the Perez sky model predicts the sky luminance values less precisely than, but close to the CIE sky model. In the absence of measured sky luminance data, sky luminance distribution predicted by the Perez sky model can be used to identify a suitable design CIE sky type for the Indian Tropical region. The steps of the framework for identification of CIE skies are as follows: (i) Relative sky luminance is predicted at 15° solar altitude through the Perez sky model using averaged monthly solar radiation of typical meteorological year data for all the 12 months in a year. (ii) The best-fit CIE sky type for each month is obtained by comparing the Perez sky model predicted values obtained in step 1 and 15 CIE sky types using root mean square error metric. (iii) Global



Figure 1. Map showing various locations chosen for the study²⁵ (with permission).

horizontal illumination is calculated for each month using the best-fit CIE sky type luminance distribution obtained in step 2 for the respective months. (iv) Best-fit CIE sky types having the minimum, second minimum and third minimum global horizontal illumination for the location are selected. (v) CIE design sky is selected from among the three sky types obtained in step 4 based on the frequency of occurrence and climatic conditions.

Using the above procedure, the design CIE sky can be identified for any location where the measured luminance data is not available. The above-defined framework was applied to the Indian tropical conditions to identify CIE design skies for major climatic zones in India.

Identification of design skies for Indian climate using the proposed framework

Typical meteorological year data for 20 cities is available in the solar radiation handbook¹⁸; therefore CIE design skies are identified in these 20 cities. Among these 20 cities, 9 belonged to the composite climatic zone, 7 to warm-humid, 2 to cold and one each from hot-dry and temperate climatic zone. The locations selected for the study are shown in Figure 1. The radiation data for the typical meteorological year of all the cities were taken from the solar radiation handbook¹⁸. After the identification of best fit CIE sky types using

After the identification of best fit CIE sky types using steps 1 and 2 of the framework, horizontal illumination is obtained from the absolute luminance values. CIE sky model gives the relative luminance of a sky element as a ratio of the zenith luminance. For obtaining absolute luminance and illumination on a plane from these relative luminance values, zenith luminance is required, either measured or modelled. Due to the lack of measured zenith luminance data at Indian stations, zenith luminance prediction models were employed for the study. The following section is about the readily available zenith luminance models and the procedure followed to select a suitable luminance model.

Zenith luminance models

Dogniaux²² proposed an equation for the calculation of zenith luminance using Linke turbidity factor (T) and

solar altitude (Z_s) at the time and location under consideration. The equation is as given below:

$$L_z = (1.234T - 0.252)\tan Z_s + 0.112T - 0.0169).$$
(13)

Perez *et al.*²³ in 1990 proposed another equation for modelling of zenith luminance from diffused horizontal irradiance and sky condition parameters as given below.

$$L_{vz} = I_{\rm DHI}[f_i + g_i \cos Z_s + h'_i \exp(-3Z_s) + j_i \Delta],$$
(14)

where f_i , g_i , h'_i and j_i are the sky condition parameters which depend on the value of sky clearness and sky brightness factor. Sastri and Manmohan²⁴ studied the equations proposed by various authors for their adequacy in predicting the variability of zenith luminance with solar altitudes in the tropical climatic zone of Delhi (India). In the study, zenith luminance was measured at different solar altitudes for winter and summer months in Delhi and the predicted values were checked against the measured values. They concluded that the Dogniaux formula gives the best possible representation of the variability of zenith luminance with solar altitude in tropical climates, if the precise value of the Linke turbidity factor is used. Perez zenith luminance model was not considered in the study. To identify which of the two models predict zenith luminance in a better way, Zenith luminance modelled through Dogniaux's equation and Perez's equation were compared with the measured values. Measured values at different solar altitudes were taken from Manmohan and Sastri's study24. Hourly values of the Linke turbidity factor were taken from the solar radiation handbook. A curve between percentage error of modelled values and solar altitude for the two equations for summer and winter season is shown in Figures 2 and 3 respectively. As it is clear from the curve that the percentage error is lower in the zenith luminance values predicted through Dogniaux's equation, it is therefore preferred over Perez's equation. The large error can be attributed to an error in the value of turbidity factor as the turbidity factor of the year of measurement may differ from typical meteorological year data. The error is large at low solar altitudes, while it decreases as the solar altitude increases. At low solar altitudes, the path of light traversing through the atmosphere close to the earth increases, which increases the scattering of light rays. Therefore, the effect of turbidity factor is more pronounced at low solar altitudes giving a large error value.

Absolute luminance values were obtained from the relative luminance using the modelled zenith luminance through the Dogniaux equation. The total illumination due to all of the sky elements was obtained from the following equation:

$$E = \int_{0}^{\pi/2} \int_{0}^{2\pi} Lv(\theta, \phi) \sin \theta \cos \theta d\theta d\phi, \qquad (20)$$

where ϕ varies from 0 to 2π and θ varies from 0 to $\pi/2$.

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Here a closed-form solution cannot be obtained for the integration of the absolute luminance values; therefore numerical integration method was used for obtaining the illuminance. The 2D trapezoidal rule was employed in this study for obtaining the illuminance from the absolute luminance values.

Identification of CIE skies

Relative sky luminance distribution was predicted at 15° solar altitude using the Perez sky model for the 20 locations in all 12 months. The best-fit CIE sky types corresponding to the Perez model predicted values at 15° solar altitude were then identified through the root mean square error metric for twelve months of each location. Figure 4 shows the best-fit CIE skies at 15° solar altitude identified for different locations in the form of a heat map. The sky conditions corresponding to each sky type can be



Figure 2. Variation of percentage error in predicted zenith luminance through Perez and Dogniaux equation at different solar altitudes in summer.



Figure 3. Variation of percentage error in predicted zenith luminance through Perez and Dogniaux equation at different solar altitudes in winter.

Lasatian				Bes	st fit CIE	E sky du	iring the	e month	n of			
Location -	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Srinagar	3	12	12	12	14	14	15	3	12	12	12	12
Shillong	12	11	11	11	12	11	11	11	8	11	12	12
Delhi	11	10	10	11	10	11	12	11	11	11	11	11
Jaipur	11	11	10	11	11	11	11	11	11	11	11	11
Patna	12	11	11	11	11	11	12	11	12	11	11	12
Varanasi	12	11	11	12	11	11	12	12	12	11	11	11
Ahmedabad	12	11	11	11	11	11	12	12	8	10	11	11
Bhopal	12	11	11	11	11	12	12	12	12	11	11	11
Ranchi	11	11	11	11	12	12	12	12	12	10	11	10
Nagpur	11	11	10	11	10	10	11	8	8	10	11	11
Hyderabad	12	12	11	12	11	12	12	14	12	11	12	12
Jodhpur	11	11	10	11	11	11	10	10	11	11	11	11
Bangalore	11	12	12	12	12	12	12	12	11	12	11	12
Kolkata	12	12	11	11	11	11	12	11	11	12	12	12
Mumbai	11	10	10	12	10	14	3	3	11	11	12	11
Pune	11	11	11	12	11	11	14	11	11	11	12	12
Vishakhapatnam	10	10	11	11	10	12	11	11	11	12	12	11
Goa	12	12	12	12	11	11	12	14	12	11	12	12
Chennai	11	11	8	12	11	11	11	11	11	11	11	12
Thiruvanthapuram	12	11	10	11	11	11	11	11	12	8	12	11

Figure 4. Heat map for the best-fit CIE skies corresponding to the Perez model predicted values at 15° solar altitude.

seen in Table 5. The yellow shaded boxes represent the partly cloudy and white blue sky. The green shaded boxes represent the clear skies of types 12 to 14 where the darker shade depicts more clearer sky. Red shaded boxes represent the overcast sky types. It can be seen from the heat map that the best-fit CIE skies identified for Indian cities mostly fall under the clear or partly cloudy sky category of types 10, 11 or 12 with a rare occurrence of overcast sky conditions owing to extreme weather conditions. Sky type 11 (white-blue sky) has the highest frequency of occurrence among all the sky types. It can be inferred from the results that the Indian tropical conditions have predominantly clear sky or partly cloudy sky at low solar altitude.

Global horizontal illuminance for the corresponding month and solar altitude were calculated using absolute luminance values obtained through the best-fit CIE sky and Dogniaux zenith luminance model. Table 9 shows the global horizontal illumination calculated using the corresponding best-fit CIE sky for the twelve months at different locations. Table 7 gives the least three illumination values among the twelve months for each location and Table 7 shows the corresponding values for CIE skies.

The first two rows of Table 10 show the design sky identified for the locations under cold climatic conditions. In Srinagar, the overcast sky type 3 has the minimum and the second minimum illumination among all the months. However, from Figure 4 it can be seen that sky type 3 is the best-fit CIE sky for only 2 out of 12 months and therefore is not frequent so as to be chosen as the design sky. Therefore, CIE sky type 12 having third minimum illumination was chosen as the design sky for Srinagar. Similarly in Shillong, CIE sky type 8 has the minimum illumination among the twelve months, but it is the best-fit sky for only one out of 12 months. Due to its transient occurrence, CIE sky type 11 having the second minimum illumination was chosen as the design sky for Shillong. Both cities have different design skies owing to the difference in the microclimatic conditions despite being in the same climatic zone. CIE sky type 12 was chosen as the CIE design sky for locations located in the cold climatic zone of the Himalayan region and CIE sky type 11 for the cold climatic zone of the northeastern region.

Sky type 11 was found to be predominant as a design sky in the locations with composite climate as can be seen in Table 10. Out of the 27 design skies having minimum illumination among the 12 months, 16 sky types were of type 11. So CIE sky type 11 was proposed as a design sky for locations falling under the composite climatic zone.

For the cities under the warm and humid climatic zone, CIE sky type 11 was having minimum illumination for 5 out of 7 cities. CIE sky type 11 also has a good frequency of occurrence among the best-fit CIE skies of warm and humid climatic zone. CIE sky type 11 was chosen as the CIE design sky for cities falling under the warm and humid climatic zone.

 Table 9. Global horizontal illumination using CIE sky model and Dogniaux zenith luminance at 15° solar altitude for 12 months at different locations

	Global Horizontal illumination (lux)											
Location	January	February	March	April	May	June	July	August	September	October	November	December
Srinagar	16,216	47,173	40,335	28,642	35,191	33,497	53,084	16,446	24,273	23,656	17,393	63,003
Shillong	8090	8437	8363	9751	11,187	8360	6971	8639	5364	8641	8090	8400
Delhi	18,313	15,068	14,925	20,422	19,935	28,277	36,630	27,367	23,508	18,563	15,756	18,896
Jaipur	14,187	15,004	14,804	19,671	23,946	29,221	34,750	32,395	21,535	16,646	17,299	15,034
Patna	25,005	21,251	18,827	22,380	26,723	33,775	39,944	29,838	27,295	20,732	18,801	22,602
Varanasi	22,815	18,500	17,144	21,046	23,030	27,112	37,220	29,197	27,775	18,284	17,450	20,340
Ahmedabad	11,811	10,673	10,586	12,808	13,916	14,748	23,891	18,316	8800	11,169	11,421	10,309
Bhopal	16,461	14,147	13,087	16,073	19,084	31,453	37,002	43,424	27,357	19,313	16,034	15,269
Ranchi	15,188	16,222	14,880	16,670	19,740	35,138	39,788	39,703	29,233	13,375	12,797	9692
Nagpur	11,419	11,785	10,199	12,252	10,197	11,407	14,195	8799	8609	10,684	12,809	11,975
Hyderabad	16,645	17,843	15,504	20,333	21,919	27,798	35,454	48,631	30,689	23,202	23,465	15,622
Jodhpur	11,142	10,682	9230	12,252	14,750	11,415	14,556	17,950	13,087	13,088	11,698	11,142
Bangalore	26,874	19,753	22,661	28,423	35,158	48,556	53,919	52,581	48,170	43,574	39,214	64,706
Kolkata	18,013	19,715	17,534	16,698	16,694	15,581	18,313	16,696	13,643	16,772	17,392	18,633
Mumbai	14,754	13,196	13,107	18,318	13,346	19,224	6361	7338	14,754	15,310	16,460	13,642
Pune	12,810	11,777	11,698	13,668	11,694	12,804	17,688	12,251	11,697	13,088	14,601	13,981
Vishakhapatnam	12,380	11,728	13,365	15,864	13,830	18,004	17,528	11,696	14,198	14,290	14,910	13,087
Goa	11,190	11,254	11,190	13,048	11,694	11,415	13,355	17,692	12,739	11,698	11,500	11,810
Chennai	11,977	12,876	8801	13,358	13,916	11,136	6970	15,584	11,141	11,421	13,367	12,741
Thiruvanthapuram	12,555	13,780	14,625	27,605	28,022	41,734	42,692	38,653	32,176	16,960	20,267	11,613

Table 10. Possible design skies for different locations from minimum from estimated minimum illumination

Location	Latitude	Longitude	Climatic zone NBC 2005 (ref. 6)	CIE sky type having minimum illumination	CIE sky type having second minimum illumination	CIE sky type having third minimum illumination
Srinagar	34.08	74.8	Cold	3	3	12
Shillong	25.58	91.89	Cold	8	11	12
Delhi	28.7	77.1	Composite	10	10	11
Jaipur	26.91	75.79	Composite	11	10	11
Patna	25.59	85.14	Composite	11	11	11
Varanasi	25.32	82.97	Composite	11	11	11
Ahmedabad	23.02	72.57	Composite	8	11	11
Bhopal	23.26	77.41	Composite	11	11	11
Ranchi	23.34	85.31	Composite	10	11	10
Nagpur	21.15	79.09	Composite	8	8	10
Hyderabad	17.39	78.49	Composite	11	12	12
Jodhpur	26.24	73.02	Hot and dry	10	11	11
Bangalore	12.97	77.59	Temperate	12	12	11
Kolkata	22.57	88.36	Warm humid	11	11	11
Mumbai	19.08	72.88	Warm humid	3	3	10
Pune	18.52	73.86	Warm humid	11	11	11
Vishakhapatnam	17.69	83.22	Warm humid	11	10	10
Goa	15.3	74.12	Warm humid	12	12	12
Chennai	13.08	80.27	Warm humid	11	8	11
Thiruvanthapuram	8.52	76.94	Warm humid	11	12	11

Typical meteorological year data for only Jodhpur city was available for hot and dry climatic zone in the solar radiation handbook¹⁸. CIE sky type 10 was obtained as having minimum illumination among the best-fit skies of different months for Jodhpur. CIE sky type 10 also had a good frequency of occurrence among the best fit CIE sky for the 12 months. Therefore, CIE sky type 10 was chosen as the CIE design sky for the hot and dry climatic zone. Similarly, typical meteorological year data for only Bangalore city was available for the temperate climatic zone. CIE sky type 12 was obtained as having the minimum illumination among the 12 months for the temperate climatic zone. Sky type 12 also had a good frequency of occurrence as it is the best-fit CIE sky for 9 out of the 12 months. Therefore, CIE sky type 12 was chosen as the design sky for the cities under the temperate climatic zone.

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Climatic zone	Proposed design sky
Hot and dry	CIE Sky type 11
Warm and humid	CIE Sky type 11
Composite	CIE Sky type 11
Cold Himalayan region/northeast region	CIE Sky type 12/11
Temperate	CIE Sky type 12

Table 11 shows the recommended design skies for the different climatic zones. As can be seen from the table, the design skies identified for major climatic zones of India are mostly clear sky types (CIE sky type 11 or 12). With the exception of cold Himalayan and temperate climate region, CIE sky type 11 is the most suitable CIE design sky for passive window design in the Indian tropical region. The design skies proposed in Table 11 can be used for the passive design of windows for any city in the Indian tropical climatic region by identifying the climatic zone and corresponding CIE design sky for the city.

Conclusion

The CIE model predicts the sky luminance better than the Perez model in the Indian tropical condition if one knows the correct CIE sky type. If the correct CIE sky type is unknown, one can still resort to the Perez sky model to obtain sky luminance values with lesser precision. A framework was proposed for the identification of CIE design skies for window design using irradiation data in the absence of measured luminance data. A set of CIE design skies are identified for the major climatic zones of India using the proposed framework. It was observed during the study that the sky occurrences at low solar altitudes in the Indian tropical climate are predominantly clear belonging to the range of CIE sky types 10 to 12. CIE sky type 11 is the most suitable CIE design sky for passive window design in the Indian tropical climatic region with the exception of the locations falling under cold Himalayan and temperate climatic zone region. The recommended set of CIE design skies given in Table 11 can be used for the passive design of windows for the locations falling under the respective climatic zones.

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