Approximate natural period expression for reinforced concrete tall buildings in India

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Many tall buildings are being constructed in different Indian cities to cater to the demand generated by the large number of people migrating from rural areas to urban centres. The safety of such tall buildings is ensured by designing them for dynamic loads, viz. wind and earthquake. To withstand these loads, computation of the natural period becomes essential. The current Indian seismic code IS 1893 (2016) has outlined a few empirical expressions based on different structural systems to compute the natural period. These expressions have been developed using data obtained from experiments performed on low to midrise buildings. Thus, verifying their applicability for tall structures before using them is important. To achieve this, in the present study ambient vibration testing was done on 28 reinforced concrete (RC) tall buildings in the Indian cities of Hyderabad and Mumbai, whose heights ranged from 50 to 150 m. These tests' natural periods were compared with existing Indian and international codes. Based on the comparison, a novel empirical expression of RC tall buildings is proposed here.

Keywords: Ambient vibration, dynamic loads, fundamental natural period, seismic codes, tall building.

IN the last few decades, urbanization in India has been occurring at an unimaginable pace, leading to the construction of many tall buildings (>50 m) to cater to this enormous demand. To ensure the structural safety of tall buildings, the Bureau of Indian Standards (BIS), New Delhi, has released a code for tall buildings[1]. The design of tall buildings is usually different compared to low and mid-rise buildings. Buildings attract lateral loads (earthquake and wind) in addition to gravity loads (dead and live). The response of tall buildings to the earthquake ground motion significantly varies due to their flexibility. Also, depending on the height, wind load dominates earthquake load. The behaviour of tall buildings largely depends on structural form, stiffness and mass distribution in plan and elevation. The same is exhibited by dynamic characteristics such as natural period and mode shapes. In design, the natural period is used to compute design seismic force[2] and dynamic effects due to wind[3]. In design practice, using an empirical expression for obtaining the natural period is common.

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IS 1893 has outlined separate empirical expressions of the natural period for three categories: bare frame buildings, buildings with structural walls and all other categories[2]. These expressions were initially developed in the USA as a part of the ATC3-06 project[4]. Later they were found not to match well with the California natural periods database in the US and were updated periodically. An earlier study in India too observed that such expressions adopted from other countries do not suit buildings in India due to considerable variations in construction practice[5]. A recent study has highlighted the shortcoming of the expression given in IS 1893 to predict the natural period of tall buildings[6]. Though IS 16700 does not propose a natural period approximate expression of a tall building, a draft version of the upcoming revision of the same code proposes a new approximate empirical expression that needs to be validated based on the measured natural period of tall buildings[7].

The present study is conducted to assess the applicability of the empirical expression for the natural period given in current seismic code[2] and, if found unsuitable, propose a novel empirical expression of the natural period for RC tall buildings (>50 m) in India.

Empirical expression in building codes

The natural period of a building is usually linked with the number of storeys or height of the building or height and base dimension or height of a building and certain dimensions of structural walls present in the building. Historically, eq. (1) mentioned below first appeared in ATC3-06 (ref. 4), derived based on Rayleigh's method[8]. In eq. (1) where T is the natural period (sec), h is the height of a building (ft), a is constant, b is the power of height 'h'. The distinct values of a were established based on the measured period of the buildings that responded to the 1971 San Fernando earthquake in California for reinforced concrete moment resisting frame (RC MRF) and steel moment resisting frame buildings. Over time, the values of a and b have been revised in successive codes such as SEAOC-88 (ref. 9) based on accumulating more such data in later years.

$$T = ah^b. \tag{1}$$

NEHRP-94[10] linked the natural period with the number of storeys N and recommended an alternative expression for

RC and steel MRF buildings (eq. (2)). This expression only applies to buildings having a maximum of 12 storeys and whose floor-to-floor height is at least 10 ft.

$$T = 0.1\ N,\ N \leq 12. \tag{2}$$

Following the trend of the US codes, India has adopted the period–height relationship as in eq. (1) and incorporated eq. (3) for RC bare frame buildings in earlier[11] and current[2] versions of the Indian seismic code. In eq. (3), h is the height of the building (m). Though eq. (3) is only to be used for RC buildings without infill walls, a recent amendment of the seismic code mentions that for RC structural wall (RCSW) buildings, the natural period computed by the approximate period expression for structural wall buildings should not exceed the period obtained from this expression.

For RC MRF bare frame buildings

$$T_a = 0.075h^{0.75}. \tag{3}$$

The period computed by taking input of building height (h) as well the base dimension (d) as in eq. (4) was recommended in a previous version of IS 1893 (ref. 11), which was intended for all buildings other than the bare frame. Equation (4) first appeared in ATC3-06 with a value of 0.05 when h and d were in feet, equivalent to 0.09 when the dimensions are in metres[4]. As discussed by Crowley and Pinho[12] this expression is derived from the equation for the frequency of vibration of a cantilever (considering shear deformation only), with the thickness of the wall considered more or less constant; thus, only the width or length of a building is an input parameter, as presented in eq. (5).

$$T_a = \frac{0.09h}{\sqrt{d}}, \tag{4}$$

$$T = 4\sqrt{\frac{m}{\kappa G}}\frac{H}{\sqrt{A}} = \frac{\alpha H}{\sqrt{A}} = \frac{\alpha H}{\sqrt{Dt_w}} = \frac{\alpha_1 H}{\sqrt{D}}, \tag{5}$$

where m is the mass per unit length, G the shear modulus, κ the shape factor to account for the non-uniform distribution of shear stresses, D the length of the cantilever and t_w is the thickness of cantilever. Some codes use this expression specifically for buildings with both frames and shear walls, while some use it for RC MRF with masonry infill panels, but many specify it for use with any building except moment-resisting space frames.

The current Indian seismic code has explicitly introduced a separate expression (eq. (6)) for the RC structural wall system[2]. In eq. (6) A_w is the total effective area of the walls in the first storey and can be computed using eq. (7). In eq. (7) A_{wi} is the effective cross-sectional area of the ith wall in the first storey, L_{wi} is the length of the structural wall in the first storey along the direction under consideration, and N_w is the number of walls in the considered direction. The shortcoming in eq. (7) is that often the length and thickness of the structural walls are not known to the designer at the initial stage. Hence, such an expression is a bit tedious for period computation. The period computed using eq. (6) is valid only if greater than eq. (4) and less than eq. (3). The measured period of buildings in the present study could not be compared with the natural period obtained by eq. (6) due to the unavailability of structural drawings.

$$T_a = \frac{0.075h^{0.75}}{\sqrt{A_w}} \geq \frac{0.09h}{\sqrt{d}}, \tag{6}$$

$$A_w = \sum_{i=1}^{N_w}\left[A_{wi}\left\{0.2+\left(\frac{L_{wi}}{h}\right)^2\right\}\right];\ L_{wi}/h \leq 0.9. \tag{7}$$

The proposed revision of IS 16700 introduces new approximate expressions for RC tall buildings[7]. Equation (8) is one of them for buildings with structural systems other than RC MRF. This expression is more or less in line with American, Canadian and Korean building codes (Table 1) except for different values of a. It will be interesting to compare all of these equations with a measured period.

$$T_a = 0.0672h^{0.75}. \tag{8}$$

Literature on empirical expression for *T*

Efforts to check and improve the code suggested empirical expressions are not new in India. Many past studies, based on ambient vibration, have been conducted periodically to improve the period expression of RC buildings in the country[5,6,13–15]. Few other studies from India have also highlighted the importance[16] of improving the natural period expression based on analytical studies[17–19]. Similarly, many studies have been conducted around the world in the last 30 years[20–31]. The present study focuses on a detailed examination of studies on RC buildings of heights greater than 50 m (Table 2).

Lagomarsino[32] studied about 185 Italian buildings, of which 52 were RC buildings. The study proposed an eq. (9) for RC buildings and revealed no correlation between the natural period and the direction of vibration.

$$T_a = h/55. \tag{9}$$

A team of Japanese researchers developed a database of 205 buildings collected from various Japanese institutions[33]. They conducted a study to develop an empirical expression for the first mode, torsion mode, and damping of these buildings. For Japanese buildings, an equation (eq. (10)) has been proposed to compute the natural period. Similarly, a Thai study[34] of 50 RC tall buildings in Bangkok proposed an equation (eq. (11)) for the natural period of these buildings, even though Thailand is not a seismically active region.

Table 1. Natural period expressions in international codes

Country	Building code	Structural system	Expression
USA	ASCE 7-16 (ref. 40)	RC SW	$T_a = 0.0488h^{0.75}$
Canada	NBCC[41,46,47]	For SW and other structures	$T_a = 0.05h^{0.75}$
Europe	EN 1991 1-4 (ref. 42)	RC multi-storey building, $h > 50$ m	$T_a = h/46$
Korea	KBC 2009 (ref. 39)	RC MRF	$T_a = 0.073h^{0.75}$

RC SW, Reinforced concrete structural wall; SW, Structural wall; RC MRF, Reinforced concrete moment resisting frames. h, Height of building; T_a, Natural period.

Table 2. Natural period expression proposed in the literature

Country	Authors	No. of buildings	Height range (m)	Expression
Italy	Lagomarsino[32]	52	Up to 200	$T_a = h/55$
Japan	Satake *et al.*[33]	25	12–170	$T_a = h/67 = 0.015h$
Thailand	Warnitchai[34]	50	20–210	$T_a = h/54 = 0.0185h$
Canada	Gilles and McClure[35]	27	20–195	$T_a = 0.019h$
India	Velani and Ramancharla[15]	32	46.21–146.75	$T_a = 0.013h$
India	Velani and Ramancharla[6]	19	63–146.75	$T_a = 0.009h^{1.10}$
Korea	Ha *et al.*[36]	58	24.2–305	$T_a = h/51 = 0.0196h$

$$T_a = h/67 = 0.015h, \quad (10)$$

$$T_a = h/54 = 0.0185h. \quad (11)$$

Canadian researchers[35] proposed an equation (eq. (12)) for the natural period of RCSW buildings in Montreal after full-scale testing of 27 such buildings. They reported that the existing Canadian code expression overestimates the natural period of RCSW buildings, leading to possible consideration of unreasonably low seismic design loads.

$$T_a = 0.019h, \quad (12)$$

$$T_a = h/51 = 0.0196h. \quad (13)$$

A recent Korean study proposed an equation (eq. (13)) for the natural period of tall Korean buildings by conducting an ambient vibration study of 58 RC buildings[36].

Studies by Indian researchers on tall buildings have also shown similar results[6,15]. Both studies[6,15] have commented on the earlier version of the Indian seismic code[11]; and tall building code was not released then[1]. Hence, the present study will be more valuable and relevant for the country.

In summary, the code-suggested expressions are unsuitable for tall buildings, since the response of such tall buildings differs from the low-rise and mid-rise buildings. The primary finding of this study suggests that tall buildings in Asia exhibit similar natural periods, as evidenced by the expressions proposed in the literature[15,33,34,36]. Further, in the absence of buildings with permanent sensors, developing empirical expressions based on ambient vibration tests is common in the earthquake and wind engineering disciplines. Thus, there is ample evidence to develop an empirical equation for tall buildings in India based on ambient vibration tests.

Measured period of tall buildings

In this study, 28 RC tall buildings were tested in the Indian cities of Hyderabad and Mumbai. Since tall buildings were the focus of this study, the shortest building in the database was 50.45 m, and the tallest height was 146.75 m. The number of storeys covered was 17 to 42. All the buildings surveyed had a structural wall system as their gravity and lateral load-resisting system. Due to the unavailability of drawings and limited access to occupied buildings, the exact details of partition walls, such as their location, orientation and material, were unknown. Except for one building, the rest were residential buildings. Table 3 gives the basic dimensions of all buildings, and Figure 1 shows one such sample building. The plan geometry of buildings varied from symmetric about one axis to symmetric about both axes and asymmetric. Figure 2 shows the drawings of only two representative building plans; Table 3 lists the remaining.

Ambient vibration was measured using a portable vibration sensor (IT Kyoshin, Japan). The sensor can simultaneously measure the vibration along all three directions with an accuracy of range +0.25 g to –0.25 g with resolving power 5×10^{-3} cm/sec^2. The vibration data were transferred with the help of an ethernet cable and stored in a laptop. An external power supply was required for the sensor. A single-point observation at the rooftop or maximum accessible floor level was recorded for 15–45 min. To measure the true lateral period, whenever possible, the sensor was kept near the centre of the building and readings were taken at the rate of 100 data points per second. To capture the period along the two major principal directions of the building, the sensor was aligned and levelled so that its two horizontal axes were parallel to the longitudinal and transverse directions of the building.

Table 3. Fundamental natural periods of RC tall buildings (>50 m) measured by ambient vibration

Serial no.	Building ID	Plan shape	Number of storeyes N	Building height H (m)	Dimension Longer L (m)	Dimension Shorter D (m)	Plan aspect ratio	Natural period (sec) Along L	Natural period (sec) Along D
1	HYB39	Rectangle	17	50.81	45.82	42.75	1.07	0.738	0.620
2	HYB44	Rectangle	17	51.15	27.27	27.14	1.00	0.569	0.700
3	HYB45	Rectangle	17	51.15	28.00	24.00	1.17	0.593	0.688
4	HYB46	Rectangle	17	51.15	27.27	27.14	1.00	0.630	0.688
5	HYB47	Rectangle	17	51.15	27.27	27.14	1.00	0.625	0.682
6	HYB51	Rectangle	17	51.15	27.27	27.13	1.01	0.616	0.645
7	HYB52	Rectangle	17	51.15	40.53	28.00	1.45	0.569	0.650
8	HYB43	Plus	17	52.98	43.11	40.38	1.07	0.751	0.694
9	MUM05	L	20	58.60	30.74	19.91	1.54	0.987	0.811
10	MUM02	Rectangle	21	63.00	49.07	24.80	1.98	1.154	1.137
11	HYB12	Rectangle	22	65.60	28.94	26.56	1.09	0.920	0.963
12	HYB13	Rectangle	22	65.60	44.55	28.97	1.54	0.910	0.952
13	HYB53	L	22	66.00	27.00	27.00	1.00	1.050	1.050
14	MUM14	Rectangle	22	66.00	26.40	23.30	1.13	1.204	1.365
15	HYB18	Rectangle	22	66.00	81.08	25.45	3.19	1.154	1.078
16	HYB23	Rectangle	17	66.23	67.64	24.45	2.77	1.154	0.871
17	MUM01	Rectangle	23	69.00	49.07	24.80	1.98	1.388	1.122
18	MUM15	Rectangle	25	71.86	24.67	13.63	1.81	1.545	1.107
19	MUM03	L	25	75.00	48.19	40.62	1.19	1.365	1.412
20	MUM16	Rectangle	26	77.86	37.60	16.80	2.24	1.545	1.222
21	HYB20	Rectangle	27	81.00	73.43	20.58	3.57	1.280	1.170
22	HYB32	T	26	83.60	50.46	42.31	1.19	1.122	1.138
23	HYB42	Plus	28	86.37	43.11	40.38	1.07	1.154	1.388
24	HYB19	Rectangle	24	87.14	80.26	46.03	1.74	1.241	1.204
25	MUM08	Oval	31	90.95	52.54	35.18	1.49	1.638	1.517
26	MUM06	T	37	119.60	46.39	29.72	1.56	2.340	1.780
27	MUM07	Y	37	137.70	51.54	37.85	1.36	2.642	2.340
28	HYB31	Z	42	146.75	33.34	29.50	1.13	3.033	3.033

Figure 1. Sample of tall building (HYB18) surveyed in the present study.

The raw data collected from the site were processed in the laboratory to compute the natural period of the building. A generalized MATLAB code was written for the entire computation[37]. As a first step, 15 min of undisturbed raw data were divided into 15 numbers of 1 min data. A baseline correction was performed using a standard MATLAB function developed by Hrovat[38]. A digital bandpass filter removed unwanted noise from the captured data. The cut-off frequency was selected based on the probable natural period of the building. These two processes resulted in 15 corrected acceleration time histories of 1 min data. The Fourier spectrum was generated for each time history data, and the average Fast Fourier transform (FFT) was computed from the 15 FFT data (Figure 3). This step ensured the removal of unwanted noise, which could not have been removed in the filtering process. From this, the average FFT fundamental natural period of the building was identified based on the power spectrum peak picking method in one direction. A similar operation was carried out for the other lateral direction. Table 3 shows the natural period identified for all the buildings using this procedure.

Proposed expression

As discussed earlier, buildings tested in the present study qualify for eq. (6). However, due to the unavailability of drawings, code-based natural period for the structural wall system could not be computed. Hence, the measured period of tall buildings was compared with eq. (4) (Figure 4 *a*). This is equally important since amendment number 2 of IS 1893 has imposed lower bound (eq. (4)) and upper bound (eq. (3)) values of the natural period computed using eq. (6). The period computed by eq. (6) must be greater than that by eq. (4) and should not be greater than that computed by eq. (3). Figure 4 *a* indicates that the code-recommended period expression (eq. (4)) for other buildings underestimates the period for buildings having $h/d^{0.5}$ greater than 20. For buildings having $h/d^{0.5}$ less than 20, the code-recommended expression sometimes underestimates or overestimates the period value. Another challenge with period expression having a lateral dimension of the building as input is that the natural period measured shows a huge difference for two buildings with the same $h/d^{0.5}$ ratio.

Figure 4 *b* shows the IS 1893 (ref. 2) recommended upper bound value (eq. (3)) of the natural period along with the proposed expression (eq. (8)) for tall buildings with other structural systems. The proposed (eq. (8)) of draft IS 16700 (ref. 7) was found to be better compared to RC bare frame (eq. (3)). However, computation of base shear from the proposed equation will give lower base shear, as it overestimates the natural period compared to the measured period. The proposed equation is found to be unconservative when it is compared with measured period data. Hence, there is a need to find an alternative equation that can reduce the gap between the actual period and the predicted period.

Figure 4 *c* shows a comparison of the measured period with other code expressions. The Korean building code (KBC)[39] expression is precisely the same as RC bare frame expression given in IS 1893 (eq. (3)). A slight difference arises due to the third digit difference in coefficient value *a* between the two expressions. Similarly, ASCE 7-16 (ref. 40) and NBCC (ref. 41) standards are nearly identical and are the most conservative of all the expressions compared in the present study. However, they too fail to predict the natural period of tall buildings in India, as they give good results only in the vicinity of a building of height 75 m. Below 75 m, they overestimate the natural period, while above 75 m they underestimate it. The natural period expression suggested by the European code for wind design[42] is different from all other expressions. This is due to the power of *h* in the expression being one/unit. The European code expression also does not match with a measured period. Thus we can conclude that natural period expression generated from the observed building periods of other countries will not be valid for tall buildings in India. Hence, it is suggested to develop empirical expressions based on the data obtained from India.

To study the influence of lateral dimensions of buildings on the fundamental natural period, the plan aspect ratios of buildings and their natural periods along both directions are plotted in Figure 5 *a* and *b* respectively. Figure 5 *a* indicates that except for four buildings, the rest have a plan aspect ratio of less than two. This could be a trend for tall buildings in India, and the same can be verified in future by collecting more such data. Despite having an aspect ratio

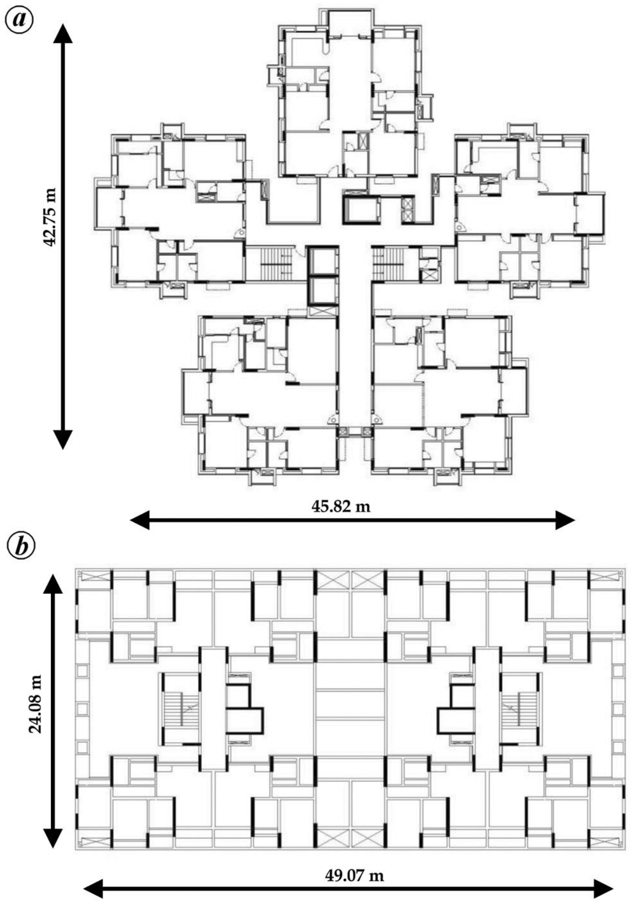

Figure 2. Sample plans of the buildings surveyed. ***a***, HYB39; ***b***, MUM01.

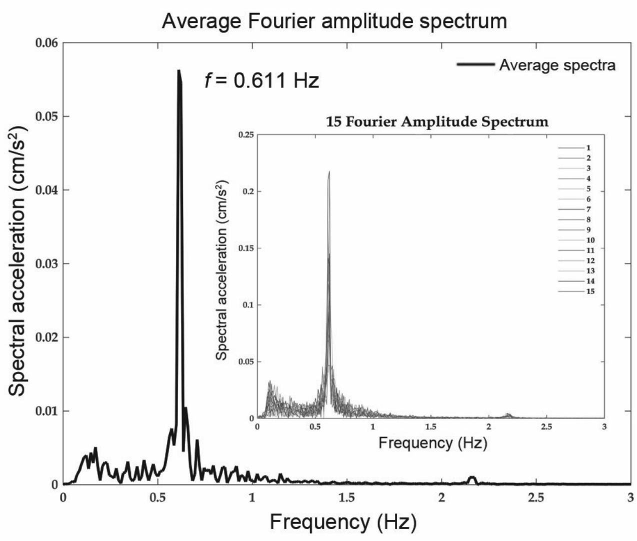

Figure 3. Mean spectra of sample building.

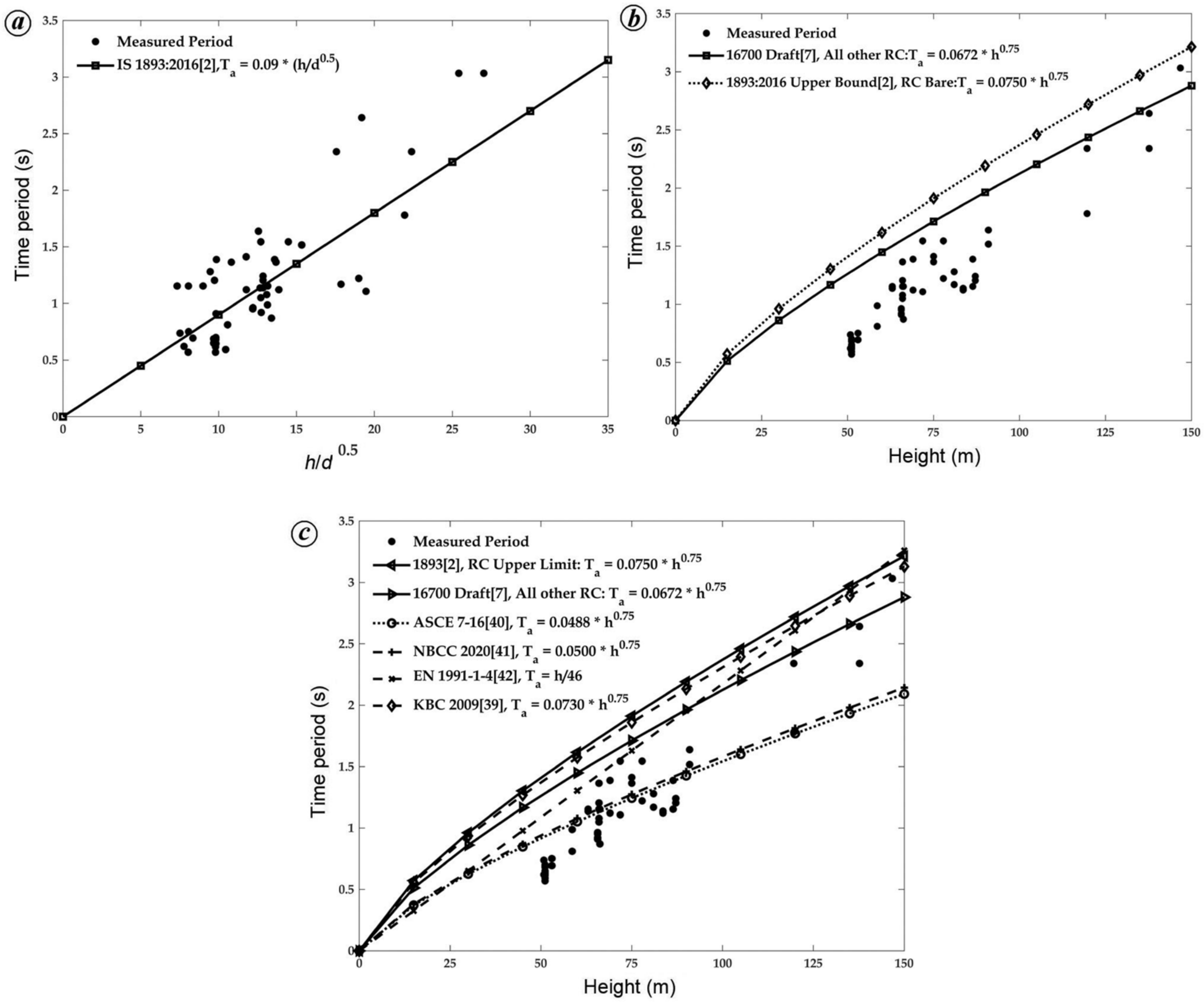

Figure 4. Measured period compared with (***a***) IS 1893 (ref. 2) expression for other structures. ***b***, RC bare frame expression of IS 1893 (ref. 2) and all other structural systems of draft revision[7] and (***c***) with few International standards.

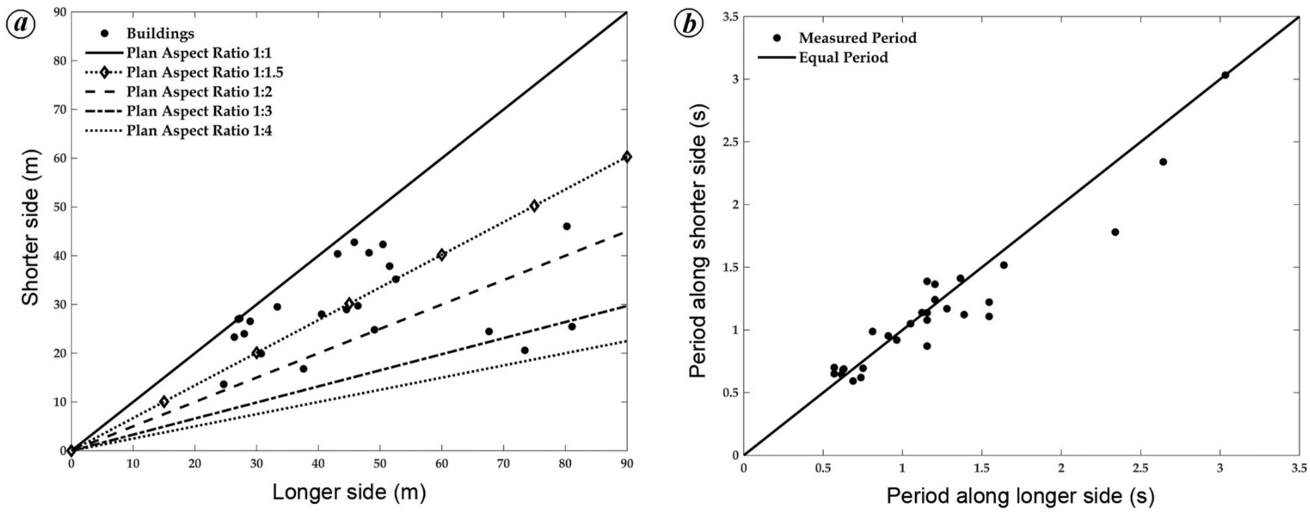

Figure 5. Influence of lateral dimensions: ***a***, Plan aspect ratios of buildings under consideration. ***b***, Relation between measured period along two principal lateral dimensions.

of more than one, the natural periods of most of the buildings are close to each other (Figure 5 *b*). This indicates that height plays an important role in the natural period, and the influence of a lateral dimension of buildings on the natural period is relatively less. This observation is in agreement with those of previous studies[29,32,43,44]. Hence we propose a novel natural period empirical expression linked with the height of the buildings in this study.

The proposed empirical expressions and the existing ones were evaluated based on statistical analysis. The equations were developed using regression analysis, and the proposed models were evaluated based on the standard error of estimate S_e (eq. (14)) and coefficient of determination R^2 (eq. (15)). S_e measure the accuracy of the prediction made by a regression model, and for a very large value of data points, it approaches the standard deviation of the measured periods from the best-fit equation. And R^2 is a statistical value that measures the degree of inter-relation and dependence between two variables; it ranges between zero (indicating no correlation) and one (indicating perfect correlation). Here, for sample size n, T_i and $\overline{T}_i$ are ith measured and computed natural period from the regression model respectively.

$$S_e = \sqrt{\frac{\sum(\log T_i - \log \overline{T}_i)^2}{n-2}}, \tag{14}$$

$$R^2 = 1 - \frac{n\sum(\log T_i - \log \overline{T}_i)^2}{\left(n\sum \log T_i^2\right) - (\log T_i)^2}. \tag{15}$$

The relation of the natural period based on height as the only input (eq. (1)) was explored by transforming both variables by means of a logarithm. The resulting data were plotted on a log–log scale, where a linear model was then fitted by eq. (16), where $y = \log(T_a)$ and $x = \log(h)$. The parameters a_1 and a_2 were determined by minimizing the squared error between the measured and computed periods, and then a was back-calculated from the relationship $a_1 = \log(a)$.

$$y = a_1 + a_2 x. \tag{16}$$

The stated procedure gives the values of a and b of eq. (1) to represent the best fit. However, for computing the base shear demand, the code-obtained natural period should give lower values. This is obtained by lowering the best-fit line by S_e without changing the slope (eq. (17)). Similarly, for the displacement-based design of tall buildings, the code-estimated natural period should be higher so that displacement demand will be more from the displacement spectra[45]. Hence, increasing the best-fit line by S_e without changing the slope was done (eq. (18)).

$$\log a_{\text{low}} = \log a - S_e, \tag{17}$$

$$\log a_{\text{upper}} = \log a + S_e. \tag{18}$$

To start with, unconstrained regression analysis was performed to obtain the values of a and b, as mentioned in eq. (1). The first trial gave $S_e = 0.142$ and $R^2 = 0.88$. In the second iteration, the power b was rounded to 1.35 and constrained regression led to almost similar values of S_e and R^2. In the third iteration, the power b was taken as 1, resulting in an increase in S_e and a decrease in R^2. Table 4 shows details of all three trials.

Similarly, constrained regression analysis was carried out by using a and b values of code expressions to compute S_e values. Such analyses have been carried out for building codes of India[2,7], USA[40], Canada[41,46,47], Europe[42] and Korea[39]. Among these, the ASCE 7-16 recommended expression was found to have the least S_e value. Table 5 shows the S_e value generated for all these expressions.

As discussed in the previous section and outlined in Tables 4 and 5, among all iterations, eq. (19) gave the least S_e value of 0.142 and the highest R^2 value of 0.88. Hence, for the conservative design of tall buildings, for base shear computation, eq. (20) has been proposed for the Indian tall building code IS 16700 (ref. 1). In future, if displacement-based design becomes popular in India, a designer can use eq. (21) to arrive at displacement based on design displacement spectra. All these three expressions are plotted in Figure 6 *a*.

$$T_a = 0.0035h^{1.35}, \tag{19}$$

$$T_a = 0.0030h^{1.35}, \tag{20}$$

$$T_a = 0.0040h^{1.35}. \tag{21}$$

The proposed equation has a power of 1.35 since the measured natural periods of tall buildings are elongated nonlinearly with increased height. This indicates that for the same structural wall system, the buildings tend to become flexible with increase in height. Accumulating more such data for buildings with the same and different structural systems will provide further insight into this aspect. The power b of the proposed equation does not match those reported in the literature[15,32–36], except in one study[6], which reported b = 1.10. The possible reason could be the difference in construction practices in different countries. It is interesting to note that none of the code-recommended expressions around the world had b greater than or equal to 1, except the wind code of Europe[42], which is common in the literature recommending new expressions for RC buildings[6,15,32–36]. India can proceed with this new proposal since b = 1.35 computes realistic natural periods, which will tend to give lesser base shear values than the existing standards. The proposed expression (eq. 20) is plotted with past literature in Figure 6 *b*.

Discussions and conclusion

The present study focuses on developing empirical expression of the natural period for RC tall buildings by measuring

Table 4. Results from regression analysis

Regression analysis type	Period expression			S_e	R^2
	Best fit	Best fit – 1σ	Best fit + 1σ		
Unconstrained	$T_a = 0.0034H^{1.3562}$	$T_a = 0.0029H^{1.3562}$	$T_a = 0.0039H^{1.3562}$	0.142	0.88
Constrained, b = 1.35	$T_a = 0.0035H^{1.3562}$	$T_a = 0.0030H^{1.35}$	$T_a = 0.0040H^{1.35}$	0.142	0.88
Constrained, b = 1.00	$T_a = 0.0153H$	$T_a = 0.0128H$	$T_a = 0.0183H$	0.177	0.82

Table 5. Standard error in code expressions with measured data

Country	Code	Period expression	S_e
India	IS 1893 (ref. 2)	$T_a = 0.075H^{0.75}$	0.583
India	IS 16700 Draft (ref. 7)	$T_a = 0.0672H^{0.75}$	0.482
USA	ASCE 7-16 (ref. 40)	$T_a = 0.0488H^{0.75}$	0.249
Canada	NBCC 2020 (refs 41, 46, 47)	$T_a = 0.050H^{0.75}$	0.260
Europe	EN 1991-1-4 Wind 4 (ref. 42)	$T_a = H/46$	0.399
Korea	KBC 2009 (ref. 39)	$T_a = 0.073H^{0.75}$	0.558

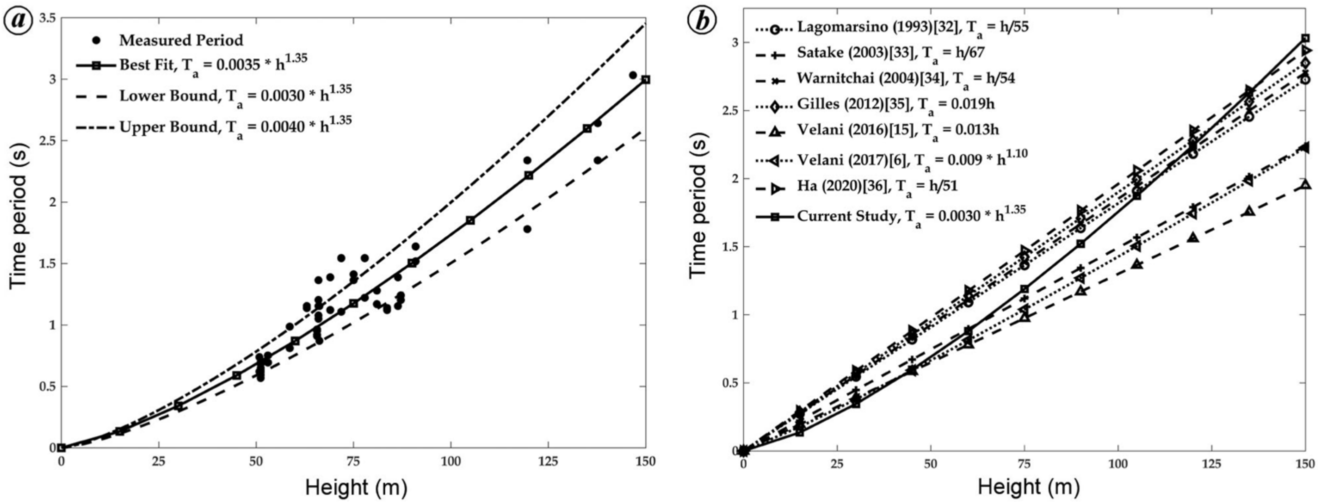

Figure 6. Proposed expression: ***a***, Best-fit curve. ***b***, Comparison with other published empirical expressions of RC tall buildings.

actual natural periods by ambient vibration test. For this, 28 RC tall buildings were surveyed in Hyderabad and Mumbai, which fall into seismic zone II and III respectively. An extensive literature survey was done to understand the current code natural period expressions around the globe and how they are being revised. Comparing the measured period with the existing international building code-recommended expressions and proposed equations of similar global studies revealed that the equations were unsuitable for tall buildings in India. A similar observation has been made while comparing the measured period with the existing Indian code expressions and the suggested expression stated in an upcoming revision of the tall building code.

Hence, the characteristics of sampled buildings were studied in detail. The lateral dimension of buildings was found to have the least influence on the natural period compared to their height. With this insight, various unconstrained and constrained regression analyses were carried out to establish the relation between natural period and height alone. Based on the standard error of estimate and coefficient of determination of various proposals, the following novel approximate expressions are proposed: (i) For force-based design to compute base shear, $T_a = 0.0030h^{1.35}$. (ii) For displacement-based design to compute target displacement, $T_a = 0.0040h^{1.35}$.

Here, T_a is the natural period (sec), and h is the height of the building (m). Force-based design expression (i) will be useful for buildings qualifying according to IS 16700 (ref. 1). For the design of code-exceeding buildings (as described by Annexure A of IS 16700)[1], the expression (ii) will be of use when buildings needs to be designed to achieve the desired performance when structure attains the target displacement during a seismic event. Using these expressions will lead to better prediction of periods, thereby computing realistic design force.

The present proposal is made based on testing 28 buildings using ambient vibration. The proposed expression in this study serves its intended purpose until a sufficient number of instrumented buildings in India are available to generate a comprehensive database of natural periods of buildings subjected to earthquake ground motions. This database will be used to refine the expression. As the period database expands to include buildings from various regions of the country, the confidence level in the expression will increase. Additionally, the validation of this expression can be carried out during actual seismic events. Looking ahead, it is recommended to periodically revise the empirical expression. Furthermore, it is suggested to develop separate empirical expressions for each structural system.

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