Quantum non-demolition measurements: concepts, theory and practice

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This is a limited overview of quantum non-demolition (QND) measurements, with brief discussions of illustrative examples meant to clarify the essential features. In a QND measurement, the predictability of a subsequent value of a precisely measured observable is maintained and any random back-action from uncertainty introduced into a non-commuting observable is avoided. The fundamental ideas, relevant theory and the conditions and scope for applicability are discussed with some examples. Precision measurements have indeed gained from developing QND measurements and some implementations in quantum optics, gravitational wave detectors and spin-magnetometry are discussed.

Keywords: Back-action evasion, gravitational waves, quantum non-demolition, standard quantum limit, squeezed light.

Introduction

PRECISION measurements on physical systems are limited by various sources of noise. Of these, limits imposed by thermal noise and quantum noise are fundamental and unavoidable. There are metrological methods developed to circumvent these limitations in specific situations of measurement. Though the thermal noise can be reduced by cryogenic techniques and some band-limiting strategies, quantum noise dictated by the uncertainty relations is universal and cannot be reduced. However, since it applies to the product of the uncertainties in non-commuting observables, there is no fundamental limit on the measurement of one of these observables at the cost of increased uncertainty and unpredictability in the other. Quantum non-demolition measurements (QNDMs) are those in which repeated measurements of the value of an observable O_1 is not hampered by quantum uncertainty generated in any other physical variable O_2 as a result of a precision measurement of O_1 (refs 1–3). One may say that a QNDM is achieved if repeated measurements of O_1 are possible with predictable results and if the back-action of the uncertainty in O_2 generated by a measurement of O_1, due to the quantum mechanical non-commutativity of the two operators corresponding to the two observables, is evaded in subsequent measurements of O_1. This class of measurement is also called back-action evading (BAE) measurement. According to an early definition by Caves[2], quantum non-demolition refers to techniques of monitoring a weak force acting on a harmonic oscillator, the force being so weak that it changes amplitude of the oscillator by an amount less than the amplitude of the zero-point fluctuations. A clearer understanding of the basic concept is immediately achieved if we examine examples cited by Braginsky *et al.*[1], especially the case of a free particle.

Consider a measurement of the position x of a particle of mass m, with a precision Δx_1. Quantum theory does not restrict this precision. However, such a measurement will introduce an uncontrolled uncertainty of $\Delta p \geq \hbar/\Delta x_1$ in the momentum of the particle. After a duration τ, the position of the particle is uncertain by $\Delta x_2 \simeq \Delta x_1 + \tau\Delta p/m$, which could be much larger than Δx_1. Hence there is significant back-action on the measurement of the position. Predictability of the position is demolished because of the back-action of the measurement through the momentum uncertainty. In contrast, the situation is very different for the measurement of the momentum observable, in principle. Measurement of momentum p with uncertainty Δp does introduce uncertainty $\Delta x \geq \hbar/\Delta p$ in the subsequent position of the particle, but this does not feed into the uncertainty of momentum. $\Delta p_2 = \Delta p_1$, as expected from a conserved constant of motion.

This example serves to define what a QND observable is. If the Hamiltonian of the system is denoted as $\hat{H}_s$, free evolution of the system observables $\hat{O}_i$ are given by

$$i\hbar \frac{d\hat{O}_i}{dt} = [\hat{O}_i, \hat{H}_s]. \quad (1)$$

To ensure that the uncertainty in $\hat{O}_i$ is protected inspite of spite of the fact that the uncertainty in a conjugate (non-commuting) observable $\hat{O}_j$ will be increased by a measurement of $\hat{O}_i$, we need $[\hat{O}_i, \hat{H}_s] = 0$ and this implies that $\hat{H}_s$ should not contain an observable $\hat{O}_j$ that does not commute with $\hat{O}_i$. For $\hat{H}_s = \hat{p}^2/2m$, the position $\hat{x}$ is not a QND observable, whereas $\hat{p}$ is.

I stress the caveat that we are still discussing the issue in principle and in practice the measurement of the momentum may boil down to the measurement of position against time (trajectory) and will suffer from back-action.

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One other point to emphasize is that these measurements do collapse the wave function in the usual sense of the phrase, with precision Δx, Δp, etc. and $\Delta x\Delta p \geq \hbar/2$. Therefore, QNDMs are not collapse-evading measurements. Nor are they the now popular weak measurements.

Another instructive example is that of an oscillator, which is archetypical for several kinds of real measurements. A quantum mechanical oscillator is governed by the Hamiltonian

$$H = \frac{\hat{p}^2}{2m} + \frac{1}{2}\, m\omega^2\hat{x}^2 \equiv \left(a^\dagger a + \frac{1}{2}\right)\hbar\omega. \tag{2}$$

The physical observables obey the uncertainty relation $\Delta x(\Delta p/m\omega) = \hbar/2m\omega$ with $\Delta x = \Delta p/m\omega$ in a 'coherent state'. This defines the standard quantum limit (SQL)

$$\Delta x = \Delta p/m\omega = \left(\frac{\hbar}{2m\omega}\right)^{1/2}. \tag{3}$$

Beating SQL implies squeezing of the uncertainties in one of the variables at the expense of the uncertainty in another.

The oscillator dynamics can be written in terms two corotating conjugate observables defined by

$$\hat{x} + i\hat{p}/m\omega = (2\hbar/m\omega)^{1/2}\,\hat{a} = (X_1 + \mathrm{i}X_2)\exp(-\mathrm{i}\omega t), \tag{4}$$

where the complex amplitude $(X_1 + \mathrm{i}X_2)$ is time-independent and hence a constant of motion.

$$\hat{X}_1 = \hat{x}\cos\omega t - (\hat{p}/m\omega)\sin\omega t,$$

$$\hat{X}_2 = \hat{x}\sin\omega t + (\hat{p}/m\omega)\cos\omega t, \tag{5}$$

with $\Delta\hat{X}_1\Delta\hat{X}_2 \geq \hbar/2m\omega$.

The crucial difference between the observable pair $(\hat{x}, \hat{p})$ and $(\hat{X}_1, \hat{X}_2)$, both of which obey the uncertainty relation, is that while the first pair has back-action dependence using the equation of motion through the free Hamiltonian $\hat{H}_0$ that depends quadratically on them

$$\frac{\mathrm{d}\hat{x}}{\mathrm{d}t} = -\frac{i}{\hbar}[\hat{x}, \hat{H}_0], \tag{6}$$

the second pair has both constants of motion

$$\frac{\mathrm{d}\hat{X}_i}{\mathrm{d}t} = \frac{\partial\hat{X}_i}{\partial t} - \frac{i}{\hbar}[\hat{X}_i, \hat{H}_0] = 0, \tag{7}$$

($\hat{x}$ and $\hat{p}$ are time-dependent, whereas $\hat{X}_1$ and $\hat{X}_2$ are not.) Therefore, if an interaction Hamiltonian H_I such that $[\hat{X}_1, \hat{H}] = 0$ can be designed for the measurement of $\hat{X}_1$, the observable can be measured without back-action from $\hat{X}_2$, which of course is disturbed by the measurement of $\hat{X}_1$.

What QNDM are not!

It is perhaps important to state briefly what QNDMs are not and this seems necessary in the context of some dismissive views expressed about the essential idea, possibly generated by the way some measurements try to achieve a QNDM. An early discussion about the context and definition is given by Braginsky *et al.*[1], who stressed the aspect of multiple measurements on the same physical system without introducing measurement-induced quantum uncertainty into the observable being measured. The essence of that discussion is that a QNDM aims to identify and measure a metrologically relevant variable for which deterministic predictability of its possibly time-dependent values is not demolished and obliterated by the quantum uncertainty introduced into another non-commuting variable as a result of the measurement of the first variable. In particular, the idea is very different in context from making repeated measurements of the same variable on a microscopic (atomic) quantum system, as in the measurement of the spin projection of an electron in a particular direction, which gives the same predictable result after the first unpredictable measurement. Limitations from quantum mechanics are to be considered not because the system itself is microscopic and atomic, but because the physical system, often macroscopic, is near its quantum ground state or its energy levels relevant for metrology need to be resolved below the zero-point contribution. The original context is detection of gravitational waves with resonant bar detectors, where it was necessary to devise methods to monitor displacement amplitudes less than 10^{-20} m of the end of a macroscopic mass weighing a ton or more, with measurement bandwidth of 1 kHz ($\tau \simeq 10^{-3}$ s) or so. This is comparable to the quantum zero-point motion of such a metal bar. A measurement with $\Delta x_1 \leq 10^{-20}$ m introduces uncertainty of $\Delta v \geq \hbar\tau/m\Delta x_1 \simeq 10^{-20}$ m/s, which will obliterate a reliable second measurement. 'The first measurement plus the subsequent free motion of the bar has "demolished" the possibility of making a second measurement of the same precision....' This may be contrasted with a recent criticism of QNDM[4]:

> If one already knows that the system is in a particular eigenstate of the measuring device, then, obviously, a measurement on the system will produce that eigenstate and leave the system intact. Zero information is gained from the repeated measurement. On the other hand, when the system is not in an eigenstate of the measuring device, the quantum state can be thought to collapse to one of its eigenstates... .

In that case, information is gained from the system, and the QND measurement most certainly demolishes the system. The concept of QND measurement adds nothing to the usual rules of quantum measurement, regardless of interpretation... .

As a common example of an imperfect measurement, consider photodetection... . Sure, the photon has disappeared, but if our detector indicates that we had one photon, we can always create another and get the same answer again and again, exactly like a QND measurement... . In every case, the concept of QND measurement is confusing and unnecessary. Why not demolish the term 'QND'?

Why is it that all the serious literature of QNDM is so easily dismissed? Unfortunately, what is referred to in this critical note is not QNDM at all in any of its forms. Such is the confusion in spite of clear examples is, in fact, a surprise for me, personally. However, in the context of this short review, it suffices to say that QNDM is a distinct and useful idea within the premises of standard quantum measurement practice and its conceptual strength will be assessed properly only after one manages to measure quantities that are at present impossible to measure otherwise. The need to keep the physical state undemolished in a QNDM is to monitor and measure its tiny changes due to an external interaction, with precision possibly below the standard quantum limit. Indeed, the abstract of a seminal paper[1] reads, 'some future gravitational-wave antennas will be cylinders of mass approximately 100 kg, whose end-to-end vibrations must be measured so accurately (10^{-19} cm) that they behave quantum mechanically. Moreover, the vibration amplitude must be measured over and over again without perturbing it (quantum nondemolition measurement). This contrasts with quantum chemistry, quantum optics, or atomic, nuclear, and elementary particle physics, where one usually makes measurements on an ensemble of identical objects and does not care whether any single object is perturbed or destroyed by the measurement....'

The key point is that while the measurement involves quantum mechanical constraints and limitations, like the uncertainty principle, the single physical system on which repeated measurements are to be made need not be microscopic. More importantly, the value of the physical observable is expected to change during the repeated measurement and that is precisely what is being monitored without back-action of the quantum uncertainty – there is no metrological interest in the repeated measurements of a quantity that is known to remain a constant.

Generalized QNDM

The basic idea of QNDM can be expanded in a useful way to bring in a larger class of measurements. All practical implementation of such a generalized picture of

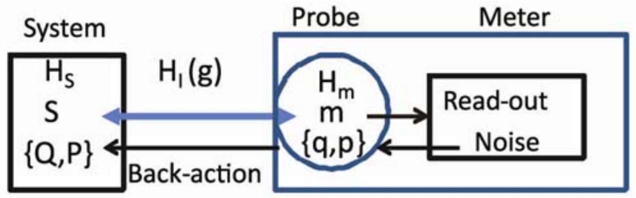

Figure 1. Scheme of a quantum measurement. See text for details. The final stage of coupling a classical meter to the probe involves collapse of the state as well as injection of quantum and other sources of noise back into the probe system. A proper choice of the probe observable avoids back-action on the signal.

QNDM involves the measurements of a system variable without significantly affecting the key observable of the system by coupling an auxiliary variable of a 'probe' system to the 'signal' such that an observable of the probe faithfully represents the signal observable (Figure 1). The probe observable is measured by a 'meter' or detector by direct interaction such that quantum disturbance created in the probe variable as a result of the measurement does not feed back into the signal, in spite of the coupling. Typically this implies that the signal and probe variables are conjugate pairs, but belonging to two different physical systems (physically both the signal and the probe may be of the same physical nature, like light). The conventional 'collapse' happens in the interaction of the probe and meter, and not in the interaction of the system and the probe. In some discussions the term 'meter' is used to refer to the probe–meter system together.

We can now write down the mathematical requirements for the definition of a QNDM. The requirement that the signal variable represented by the quantum mechanical operator $\hat{A}(t)$ is deterministically predictable implies that

$$[\hat{A}(t_j), \hat{A}(t_i)] = 0, \tag{8}$$

for different times t_k. For example, for the free particle, momentum satisfies this relation, being a constant of motion. For an oscillator, the position and momentum are

$$[\hat{x}(t), \hat{x}(t+\tau)] = \frac{\mathrm{i}\hbar}{m\omega}\sin\omega\tau,$$

$$[\hat{p}(t), \hat{p}(t+\tau)] = \mathrm{i}\hbar m\omega\sin\omega\tau. \tag{9}$$

The commutators are zero only at specific instants separated by a half-period, for each observable, and they are called stroboscopic QND variables. Labelling two non-commuting system observables as $\hat{S}_i \equiv \{\hat{Q}, \hat{P}\}$ and the probe–meter observables as $\hat{m}_j \equiv \{\hat{q}, \hat{p}\}$, with their own Hamiltonian evolutions and an interaction Hamiltonian $\hat{H}_\mathrm{I}$ for the coupling between the system and the meter

$$i\hbar\frac{\mathrm{d}\hat{S}_i}{\mathrm{d}t} = [\hat{S}_i, \hat{H}_\mathrm{s}] - [\hat{H}_\mathrm{I}, \hat{S}_i],$$

$$i\hbar \frac{d\hat{m}_j}{dt} = [\hat{m}_j, \hat{H}_m] - [\hat{H}_I, \hat{m}_j]. \tag{10}$$

While the observable $\hat{S}_i$ could be time-dependent, as in the case of the position of a mirror due to the interaction with a passing gravitational wave, QNDM demands that it does not change due to the interaction with the meter system that is used to read out the value of the variable. So, a QND observable of the system satisfies $[\hat{S}_i, \hat{H}_s] = 0$. For the same observable to be back-action evading, it should satisfy $[\hat{S}_i, \hat{H}_I] = 0$. Since we want the meter observable $\hat{m}_j$ to change due to the coupling to the system, for an efficient measurement $[\hat{H}_I, \hat{m}_j] \neq 0$. Taking the QND observable $\hat{S}_i$ as $\hat{Q}$, these requirements suggest that the meter observable for readout should be $\hat{p}$ and that the interaction Hamiltonian could be of the form

$$\hat{H}_I = g\hat{Q}\hat{q}. \tag{11}$$

The back-action from the meter is evaded by choosing the system and meter observables with a conjugate nature, like intensity of the signal beam and phase of the meter beam in an optical QNDM. For example, in an optical QNDM, the system observable could be the intensity and the phase of the probe beam the readout observable, with an interaction Hamiltonian $\hat{H}_I = \chi \hat{n}_s \hat{n}_p$, where χ is the optical Kerr nonlinearity. For the measurement to qualify as a 'good' measurement, the correlation between the variations in the signal and the probe has to be high enough, ideally unity. This is achieved by choosing the right Hamiltonian and the coupling g, keeping in mind that the choice is constrained by the need to evade back-action.

Demonstrations

Several demonstrations of QNDM are now available, mainly in the context of quantum noise-limited measurements in several areas of optics and atomic physics. There have been some demonstrations that are in tune with the development of original ideas in QNDM, for macroscopic mechanical systems observed close to their quantum ground state where quantum noise is readily observable. We mention a limited sample to clarify the essential concepts. However, we omit the details of implementation and analysis and refer to the relevant papers for details.

Opto-mechanical system

In this example, the metrological goal is to monitor the quantum radiation pressure noise of an optical signal beam by its mechanical effect on the position of macroscopic mass attached to a spring, forming a classical oscillator (or a quantum oscillator with extremely small spacing in the quantized energy). A natural choice for the meter is another weak optical beam. The coupling between the signal and meter is achieved by the device of an optical cavity with which both light fields are resonant (Figure 2). The macroscopic mass oscillator is one of the mirrors of the cavity in the QNDM implementation[5,6]. Then the intensity fluctuations of the signal, either due to a modulation or due to quantum fluctuations (radiation noise pressure), will translate to displacement noise of the mirror. However, since the meter field is resonant with the cavity, the intensity of the reflected field is unaffected to first order in displacement, but the phase of the meter beam (with weak intensity) is linearly affected. This enables a faithful measurement of the signal beam intensity variations, without any back-action on the intensity of the signal beam, through the signal obtained by forming an optical cavity with the oscillator mass as one of the mirrors. The physical system itself resembles closely the configurations in interferometric gravitational wave detectors, where the actual signal is the displacement x of the mirror that is measured as first-order phase changes in the probe light, but affected by the radiation pressure noise through the interaction Hamiltonian of the form $H_I = g\hat{n}\hat{x}$, where $\hat{n}$ is the photon number operator. (Interaction Hamiltonian of the form $H_I = \lambda\hat{F}\hat{x}$ is generic for measurement of weak forces.)

The coupling between the signal and probe beams has been implemented in several experiments employing the nonlinear optical effects inside the cavity.

Successful implementations are considerably more complicated, done at cryogenic temperature, involving Hamiltonians nonlinear in the observables[7] (in contrast to bilinear Hamiltonians with coupling coefficients representing a nonlinearity). The most important metrological context for optomechanical QNDM is the detection of gravitational waves with advanced optical interferometers, which I will discuss later.

Optical QNDM and the quantum tap

As in other QNDM schemes in practice, optical QNDM couples a meter beam to a signal beam, typically through an atomic medium and then the strong correlation between the meter observable and the signal observable is used for a measurement of the signal by a real measurement on the meter beam[8–10]. The observables are chosen

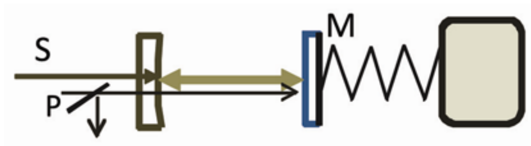

Figure 2. Radiation pressure of the signal beam (S) causes fluctuations in the position of the mirror on spring (M) and in turn changes the phase of the resonant weak probe beam (P) in the cavity configuration.

such that there is no back-action. Optical QNDM makes use of nonlinear interaction between a signal beam and a meter beam, through a generalized Kerr effect – intensity dependent changes in the effective refractive index, $n = n_0 + n_2 I$, due to the presence of the optical beam with intensity I. This is characterized by a nonlinear phase shift proportional to intensity

$$\phi_i = \frac{2\pi l_i}{\lambda_i} n_{2i} I_i, \qquad (12)$$

where the index refers to either 's' or 'm', signal beam or meter beam. The cross-gain for the coupled system is

$$g = 2\sqrt{\phi_\mathrm{m}\phi_\mathrm{s}}, \qquad (13)$$

which defines how the fluctuations in one beam feed into the other. Denoting the fluctuations in amplitude and phase quadratures as δX and δY, we have

$$\delta X_o^\mathrm{s} = \delta X_i^\mathrm{s} \qquad \delta Y_o^\mathrm{s} = \delta Y_i^\mathrm{s} - g\delta X_i^\mathrm{m},$$

$$\delta X_o^\mathrm{m} = \delta X_i^\mathrm{m} \qquad \delta Y_o^\mathrm{m} = \delta Y_i^\mathrm{m} - g\delta X_i^\mathrm{s}, \qquad (14)$$

because the two intensities are decoupled but the phases are coupled. The amplitude quadrature fluctuation is $\delta X = \delta n/\sqrt{n}$ and the phase quadrature is $\delta Y = 2\delta\phi\sqrt{n}$, where n is the number of photons.

Since the intensity variations cause only a change in the phase and not the intensity of the coupled beam, back-action is evaded. The modulations of the signal beam can be measured as modulations of the phase of the meter beam without affecting the intensity of the signal beam. Though the intensity noise in the meter beam does affect the phase of the signal beam, it does not feed into the other quadrature that is being monitored. Naturally, an interferometric set-up in which the phase of the meter beam is measured with reference to the stable reference of a split-off part of the meter beam is required (Figure 3).

Criteria for an optical QNDM have been developed and discussed in the literature[8,11,12]. Since quantum noise is unavoidable, one usually has $\Delta X_\mathrm{s}\Delta X_\mathrm{m} \geq 1$, with the equality achieved at SQL. A QNDM is characterized by $\Delta X_\mathrm{s}\Delta X_\mathrm{m} < 1$. Defining the signal-to-noise ratio as $R = \langle X\rangle^2/\langle\delta X\rangle^2$ for the various beams, the goal is to minimize additional noise in the interaction of the signal and meter such that the transfer function for R from input to output ($T = R_\mathrm{out}/R_\mathrm{in}$) is as close as possible to unity. For an ideal classical beam splitter (a classical optical tap), for example, with transmissivity t^2, $X_\mathrm{out}^\mathrm{s} = t^2 X_\mathrm{in}^\mathrm{s}$ and the meter output will have the rest of the signal beam, $(1 - t^2)X_\mathrm{in}^\mathrm{s}$. Hence $T_\mathrm{s} + T_\mathrm{m} = 1$ and no classical device can exceed this. However, $\Delta X_\mathrm{s}\Delta X_\mathrm{m} < 1$ implies $T_\mathrm{s} + T_\mathrm{m} \geq 1$ and ideal QND can approach $T_\mathrm{s} + T_\mathrm{m} = 2$. One implementation of these ideas, with $T_\mathrm{s} + T_\mathrm{m} > 1$, was realized with the nonlinear coupling generated using a three-level atom in which the ground state is coupled to the strong transition by the detuned weak probe beam and level 2 to 3 in the ladder by the strong signal beam[9,13]. This scheme avoids absorption from the signal beam, yet preserving the strong coupling between the signal and the probe, providing a phase shift of the probe proportional to the intensity of the signal beam. Intensity of the signal beam is not affected by the increased uncertainty in the amplitude quadrature of the probe due to the precision phase measurement because it changes only the phase of the signal and not its amplitude, again through the Kerr coupling, enabling back-action evasion.

Atomic spin systems

Atomic spin systems offer a metrologically important physical scenario for implementing and testing QNDM schemes[14,15]. For individual atomic spins the projections alone different directions are non-commuting observables. For a spin ensemble, with total spin $S = (S_i, S_j, S_k)$

$$\langle\Delta S_i^2\rangle\langle\Delta S_j^2\rangle \geqslant \frac{1}{4}\langle\Delta S_k^2\rangle. \qquad (15)$$

A coherent spin state is one that satisfies the minimum uncertainty with equal uncertainties in the two directions. Therefore, the spin state is considered squeezed when one of the uncertainties $\langle\Delta S_\mathrm{i}\rangle < (1/2)\langle S_k\rangle$. This is consistent with the idea that for a spin system polarized along a particular direction, the spin noise (variance) scales as the number of spins N. The elementary spin being $\hbar/2$ with variance $\hbar^2/4$, the spin S is worth $2N$ elementary spins and hence the variance of uncorrelated spins is $S/2$. Squeezing then involves generating correlations among the elementary spins by an interaction. A measurement of one projection with a precision $\langle\Delta S_x\rangle < (1/2)\langle S_k\rangle$ results in a spin-squeezed state with increased uncertainties in the other projections (a weaker condition $\langle\Delta S_x\rangle < (1/2)\langle S\rangle$ was

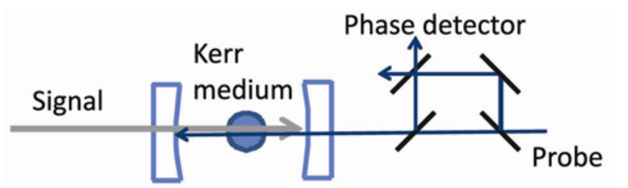

Figure 3. Schematic diagram of an optical QNDM. The strong signal and weak probe beams interact via a Kerr nonlinearity in the atomic medium, causing a change in the phase of the probe proportional to the intensity of the signal. Intensity of the signal beam is not affected by the increased uncertainty in the amplitude quadrature of the probe because the precision phase measurement changes only the phase of the signal and not its amplitude, enabling back-action evasion.

shown to be sufficient for increased bandwidth of measurements at the quantum limit[16]). The conditions on the Hamiltonian of the system S and the probe m are obvious

$$[S_z, H_S] = 0; \text{ ensures } [S_z(t_2), S_z(t_1)] = 0,$$

$$[S_z, H_I] = 0; \text{ ensures BAE},$$

$$[s_z(m), H_I] \neq 0; \text{ ensures that } s_z(m) \text{ is a valid probe}, \quad (16)$$

and this suggests $H_I = \alpha S_z s_z(m)$.

Precision magnetometry with sensitivity reaching a femto-Tesla is a motivating factor for QNDM on spin ensembles. The fundamental noise is the quantum spin shot noise with SQL variance of $S/2$ for the spin-S ensemble. The basic measurement scheme involves the Larmour precession of the spins in a weak magnetic field which can modulate the polarization of a weak linearly polarized probe beam that is detuned from the hyperfine resonances. With no net polarization, one obtains a polarimetric signal of the quantum noise at the Larmour frequency[16,17]. The goal is to implement a QNDM of a magnetometer signal, which is the Larmour precession of the coherent polarization generated in the atomic vapour with a circularly polarized pump beam. Implementation of QND measurement with a stroboscopic BAE scheme in atomic vapour of potassium is discussed by Shah *et al.*[15].

QNDM of photon number in a cavity

An impressive application of the QND idea that goes beyond demonstration of principles and strategies is that of the measurement of the number of photons inside a high finesse optical cavity, without altering this number by absorption, by observation of the change in the phase of atomic states of a passing atomic beam that interacts with the photons inside the cavity[18]. The Stark shift (light shift)-induced splitting of the energy levels of the atom in the cavity containing n photons (obtainable from the Jaynes–Cummings model) is

$$\Delta E = \frac{\hbar\Omega^2}{2\Delta}n + \frac{\hbar\Omega^2}{4\Delta}, \quad (17a)$$

which results in an n-dependent discrete phase shift,

$$\Phi_c(n) = \frac{\Omega^2\tau}{2\Delta}n, \quad (17b)$$

the experiment is implemented as a Ramsey interferometer with three microwave cavities, with the two auxiliary cavities for state preparation and analysis with a precisely tunable phase difference between them (Figure 4). A $\pi/2$ pulse of microwave radiation is applied in the first cavity to atoms prepared in the excited state, which changes the state to a coherent superposition of the ground and excited states. The state will evolve due to free evolution as well as due to the phase acquired in the cavity. The final state of the atoms (e or g) is detected after a second $\pi/2$ pulse in the final cavity with tunable Ramsey phase Φ. Scanning the Ramsey phase results in sinusoidal modulation of the average fraction of the two atomic states and of the probability of detection in a particular state (Figure 5). For example

$$P_{|e\rangle\to|g\rangle} = \cos^2\left(\frac{\Phi_c(n)+\Phi}{2}\right). \quad (18)$$

There are two observables, atom in the ground state and atom in the excited state, which are complementary. Since the probability depends on the discrete number of photons in the main cavity, the sinusoidal probability curve will shift in phase by a discrete jump when one photon is added or subtracted from the cavity. Therefore, each set of measurements of $P|e\rangle\to|g\rangle$ or $P|e\rangle\to|e\rangle$ determines the photon number probabilistically.

If the atom prepared in a excited state comes out in excited state after the interaction with the cavity, then its phase is shifted by 0 or 2π and the photon number in the cavity is most probably 0 or n, with sinusoidal variation

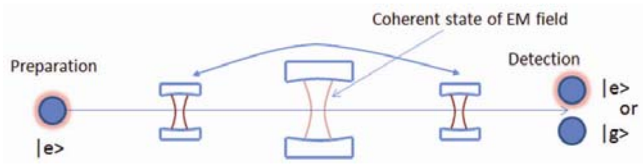

Figure 4. QND–BAE measurement of the number of microwave quanta in the cavity. The central cavity has a small number of photons that change the relative phase of the superposition of the excited and ground states of the passing atoms. The two auxiliary cavities define a Ramsey interferometer with a scannable relative phase. Final state selective detection enables an iterative determination of the number state inside the main cavity. See text for more details.

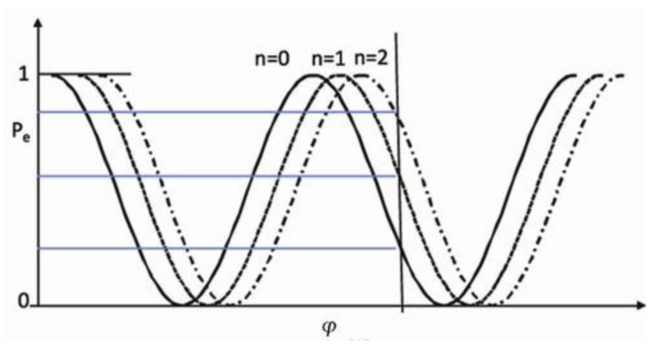

Figure 5. The probability to get a particular final state as a function of the Ramsey phase. The three curves are for three different photon numbers inside the cavity.

of the probability for other photons numbers (the interaction is tuned to get a particular predetermined phase shift of 2π for n photons). If the atom is detected in the ground state, the phase is $\pi/2$ and the probability peaks at photons number $n/2$. Since the detuning is large, only the phase of the atoms is affected and there is no photon absorption or stimulated emission, maintaining the QND nature of the measurement. The interaction with the atoms does feed back to the phase of the cavity field, but that does have any back-action on the photon number.

In this example, the observables do not return definite values, but only a probability distribution. The measurement is characterized as a two-element POVM (positive operator valued measure) S_j corresponding to the state of the detected atom ($S_0 + S_1 = I$), which in turn determines a partial (probabilistic) measurement of the photon number ($\hat{n} = a^{\dagger}a$) in the cavity.

$$S_j = \cos^2\left(\frac{\Phi + \Phi(a^{\dagger}a) - j\pi}{2}\right). \quad (19)$$

If ρ is the initial state of the field, the probability of finding the atom in state j is

$$P_j(\rho_i) = \mathrm{Tr}(\rho S_j) \quad (20)$$

A detection of the atom in state j projects the field state to

$$p_p(j) = \frac{\sqrt{S}_j \rho \sqrt{S}_j}{\mathrm{Tr}(\rho S_j)}. \quad (21)$$

One is effectively starting with a uniform initial density matrix (probability being equal for photon numbers from 0 to n) and then building up $\rho_p(j)$ in repeated QNDMs. This is one case where repeated measurement without demolition of the state is achieved with new information gained in each step of the experiment, providing a strong counter example to the criticism expressed by Monroe[4].

Braginsky[19], who is one of the originators of the QNDM idea, remarked about these measurements:

> Several years ago, S. Haroche and his colleagues successfully demonstrated absorption-free counting of microwave quanta. In my opinion, this is one of the most outstanding experiments conducted during the second half of the 20th century.

Squeezed light in gravitational wave detection

Since the focus has now shifted from resonant metal oscillator detectors to optical interferometers for the detection of gravitational waves, beating the standard quantum limit for measurements also is focused in the optical domain, specifically in the use of quantum noise-squeezed light and its vacuum state. Indeed this direction of research has turned out to be successful in practical terms for the gravitational wave (GW) detector, and the advanced interferometer detectors that are being commissioned for observations have been tested with squeezed light, with promising benefits in sensitivity and stability of operation. Referring back to our discussion on QND with a mechanical oscillator and light, we can sketch the basic idea. The gravitational wave causes small oscillations of the suspended mirrors of the optical cavity and this causes first-order changes in the phase of the stored light and only second-order changes in its intensity (being locked to the peak of a Fabry–Perot resonance). Hence the gravitational wave signal is in the phase quadrature, contaminated by the minimum uncertainty noise in the same quadrature of the coherent state vacuum. The noise in the intensity quadrature is radiation pressure noise that affects the position of the mirror, causing additional noise in the phase quadrature, if large. The detection shot noise in the phase quadrature relevant for the interferometer sensitivity decreases as $\bar{n}^{-1/2}$, where $\bar{n}$ is the average number of photons in the detection band, whereas radiation pressure noise on the mirror is the fluctuation in the momentum transfer ($p = 2\bar{n}h\nu/c\,h$ and increases as $\bar{n}^{1/2}$. The two variances add and determine the SQL. However, the radiation pressure noise is frequency-dependent when translated into the actual mirror motion because the mirrors are suspended as pendula and the response decreases as $1/f^2$, where f is the natural frequency.

In the real situation, the radiation pressure noise is significant only at low frequencies (below 50 Hz or so) and the photon shot noise dominates the high-frequency region of the detection band. The physical picture is that

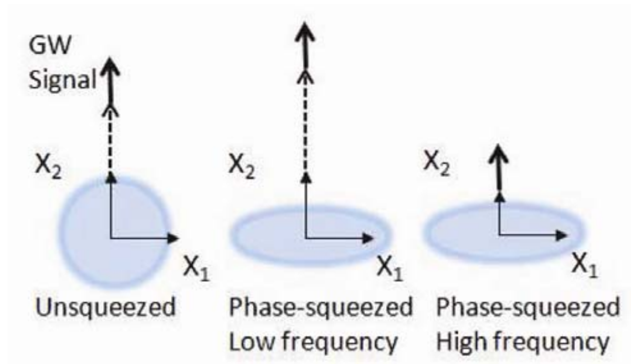

Figure 6. Scheme of noise reduction by squeezing the vacuum noise, shown here for squeezing in the phase quadrature. The GW signal is in the phase quadrature (X_2) and its measurement is limited by the quantum shot noise as well as the radiation pressure noise (dotted arrow). Squeezing the phase quadrature reduces phase noise and improves that sensitivity to GW, but it also increases the radiation pressure noise because the amplitude (X_1) uncertainty increases (back-action). This extra noise is avoided at high frequency because of the mirror pendulum response, but it limits sensitivity at low frequency. So sensitivity below shot noise is achieved at high frequency (adapted from Virgo-Ego Scientific Forum 2012 Summer School lecture slides by Stefan Hild, University of Glasgow, UK).

the vacuum noise enters the output port of the interferometer and adds to the gravitational wave signal in the phase quadrature. Hence, any squeezing of the phase quadrature, at the expense of increased noise in the amplitude quadrature, reduces noise in the high-frequency detection band where back-action from the amplitude quadrature through radiation pressure noise on the mirror is insignificant (Figure 6). This is then equivalent to the use of higher laser power (more photons) in the interferometer, reducing the quantum shot noise. However at low frequencies, the increased noise in the amplitude quadrature will cause increased noise for gravitational wave detection. This can be avoided only by frequency-dependent squeezing, where the phase quadrature is squeezed at high frequencies and amplitude quadrature is squeezed at low frequencies. Implementation of sensitivity significantly below shot noise in the relevant detection band is yet to be demonstrated in full-scale GW detectors, but feasibility has been demonstrated in these very detectors at high frequency[20,21].

Renewed relevance of QNDM

The efforts to detect gravitational waves have shifted focus from cryo-cooled resonant detectors to interferometer-based detectors with free mirrors as the sensing masses. In such detectors, the expected displacement of the masses is less than 10^{-19} m, which is smaller than the quantum zero-point motion of these suspended mirrors. More seriously, the thermal motion is over a million times larger, unlike in the cryo-cooled bar detectors where residual thermal and quantum motions are comparable. However, effective metrology is possible because the pendular suspensions of the mirrors have very high Q (quality factor), and nearly the entire thermal and quantum energies are concentrated at the oscillation frequency of about 1 Hz. Non-dissipative feedback techniques are used to keep these motions within certain limits and the actual detection bandwidth starts 20–30 times higher in frequency where the residuals from the quantum and thermal motions are below the levels that can affect the measurement. So, there is a clear separation between the detection bandwidth and resonance bandwidth, in contrast to the resonant detectors where both merge. Since resonant bar GW detector was the only metrological scenario that necessarily needed a QND–BAE measurement for its success when these ideas originated, one may wonder about the relevance of such ideas in the context of advanced interferometer detectors. However, as we have seen, the interferometric measurement is also limited by quantum noise in the optical phase and amplitude quadratures and QND techniques with squeezed light are turning out to be essential for the efficient operation of such detectors. Also, QND metrology may significantly improve sensitivity and bandwidth in magnetometry and rotation sensing (atomic gyroscopes) with spin-polarized atomic ensembles. Another area of application where QNDM is indispensable is in feedback cooling of macroscopic oscillators to their quantum ground state[7,22], which requires back-action evading measurements for noise-free feedback.

Summary remarks

A survey of the experimental implementations of quantum non-demolition measurements with back-action evasion, nearly four decades after such ideas were first proposed, suggests that QNDM is maturely understood and has been demonstrated in multiple physical systems. QNDM is demonstrated to be a useful, superior tool in those situations where metrology has to be done close to the quantum noise level. Implementations are now a growing list, including high-precision magnetometry and several types of optical measurements. QNDM is crucially useful when not even measurements at the standard quantum limit can take one to the goal of the measurement, as in the gravitational wave detectors. Squeezed light technology as implemented in optical interferometers may prove to be the single-most important technology push that is required to usher in gravitational wave astronomy.

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ACKNOWLEDGEMENTS. The invitation from Prof. N. D. Hari Dass and other organizers to the Disqussions Meeting on Quantum Measurements was an important opportunity for me to get more familiar with quantum non-demolition measurements, which I believe will play a significant role in future precision metrology below conventional quantum limit. However, I do not think that this brief review does justice to the vast amount of published work by several researchers, but I hope it will serve as a useful pointer.

doi: 10.18520/v109/i11/2052-2060