Puzzles

Compiled by
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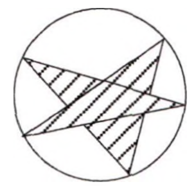
Q.1. Find the sum of all numbers greater than 10,000 formed by using the digits 0, 2, 4, 6, 8, no digit being repeated in any number.

Q.2. Every man who has lived on earth has made a certain number of handshakes. Prove that the number of men who have made an odd number of handshakes is even.

Q.3. In a group of 10 persons, each person is asked to write the sum of ages of all other 9 persons. If all the 10 sums form the nine-element set {82, 83, 84, 85, 87, 89, 90, 91, 92}, find the individual ages assuming them to be natural numbers.

Q.4. A regular five pointed star is inscribed in a circle of radius r (see the figure). Find the area of the region inside the star.



Q.5. Find the number of prime numbers.

Q.6. There are thousand closed doors numbered from 1 to 1000. Thousand persons serially attend the doors. 1st person will attend all the doors and open them as they are closed. 2nd person will attend 2nd, 4th, 6th, ... doors and closed them as they are open (opened by 1st), 3rd person will attend 3rd, 6th, 9th, 12th, ... doors and closed the 3rd, 9th, ... as they are open (opened by 1st) and open 6th, 12th, ... doors as they are closed (closed by 2nd person). In this fashion each person will be passed. After passing the last person, how many doors will remain opened?

Q.7. A crocodile is known to have not more than 68 teeth. What is the maximum number of crocodiles having different set of teeth?

Q.8. If "P' is a prime number then prove that $n^P - n$ is divisible by P for any integer n.

Q.9. John has x children by his first wife. Mary has x + 1 children by her first husband. They marry and have children of their own. The whole family has 24 children. Assuming that two children of same parent do not fight, find the maximum possible number of fight that can take place.

Q.10. Prove that $1^k + 2^k + \ldots\ldots + n^k$, where n is an arbitrary integer and k is odd, is divisible by $1+2+\ldots\ldots+n$.

Answers :

1. 519960
3. 15, 14, 13, 12, 10, 8, 7, 7, 6, 5
4. $\dfrac{10r^2 \tan(\pi/10)}{3-\tan^2(\pi/10)}$
5. Infinite
6. $[\sqrt{1000}] = 31$
7. 16^{17}
9. 191

* Final Year, CST.