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TWO DIMENSIONAL GRID TRANSFORMATION BY ALGEBRAIC METHOD

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Abstract: Grid transformation in systematic form is used to smooth the meshing process. Through proper steps an orthogonal grid system of the unit square is applied for simulation. The basic grid conversion method suggests mainly constructing explicitly coordinate changes with the help of transfinite interpolation. In this regard, blending functions are quite important. These apply matching of the grid arrangements at the boundaries and for different interior surfaces of a particular domain. The aim of this present work is to explain specific techniques of algebraic grid transformation through transfinite algorithm in 2 dimensional spaces. Few irregular but structured domains are converted into regular structured domains through transfinite algorithm to apply them in different fields.

Keywords: Structured grid, algebraic transformation, transfinite algorithm, interpolation.

1. INTRODUCTION

The necessity of grid transformation method is to increase the efficacy of the solution for difficult situations. In fact, the grid is the smallest zone of interest in a specific domain on which continuous differential equations are changed into discrete finite difference equations to analyze a problem through different but case specific codes.

Liou and Jeng [1] proposed an algebraic grid generation technique based on Soni-Linear and Soni-Hermite algorithms. In Soni-Linear method, two initial grids are formed from to opposite faces and then they are coupled to form final grids. Soni-Hermite method is an interpolation method of data points to generate polynomial functions. Conti et al. [2] developed an algorithm to generate boundary orthogonal grids with coupling of mixed Hermite algebraic method with a boundary based orthogonal schemes. Backman and Huynh [3] used transfinite algorithm to determine maximum flow in a network domain. Eriksson [4] proposed a technique to apply transfinite algorithm in 3 dimensional surface-coupled meshing types with non-orthogonal nature. Allen [5] developed a volume evaluation method in the form of transfinite interpolation. Many other researchers developed many codes regarding grid generation. But most of the methods are very complex algorithm based and very much case specific.

In this present work, an attempt is made to apply transfinite algorithm to transform structured but irregular grids into structured regular grids so as to minimize the computation time and also to fit the boundary conditions with ease. This transfinite algorithm method is easy to understand compared to other transformation methods and equations can be explicitly applied in this method [6].

2. ALGORITHMIC APPROACH

It is required to transform structured irregular grids in *xy* plane to structured regular grids in *qp* plane.

q-constant and *p*-constant lines are first created in the required domain and then they are converted into 1x1 orthogonal grid system.

The algorithmic steps required for the grid transformations are as follows:

1) The grid points must be placed on the boundaries. The number of points on opposite faces should be equal to form a rectangular domain.
2) On a particular *q*-constant line equi-spaced points are generated by interpolation along *p* direction. Repeat the similar process for every *q*-constant line containing both the boundaries ($q = 0$ and $q = 1$).
3) There may be some cases where the grid points generated by the above stated process do not coincide with existing grid points at boundaries. To correct this, the difference of the existing points and the derived points are evaluated. This deviation then is subtracted on the $q = 0$ and $q = 1$ lines.

3. CASE STUDIES

3.1 Case I

Let the irregular domain as shown in the Fig.1(a) is taken. Its faces EF, FG, GH, HE are represented by equations $f_1(x,y) = 0$, $f_2(x,y) = 0$, $f_3(x,y) = 0$ and $f_4(x,y) = 0$ respectively. The equations of the four faces are-

$$f_1(x,y) = y = 0 \tag{1}$$

$$f_2(x,y) = 4-y-2x = 0 \tag{2}$$

$$f_3(x,y) = 2y-x-2 = 0 \tag{3}$$

$$f_4(x,y) = y-2x = 0 \tag{4}$$

By using the algebraic method described earlier,

$$q = y/(2+x-y) \tag{5}$$

$$p = (y-2x)/(2y-4) \tag{6}$$

By inverse relationships,

$$x = 4p(q+1)-2q(2p-1)/(2p+q+2) \tag{7}$$

$$y = 4q(1+p)/(2pq+q-2) \tag{8}$$

By employing transfinite algorithm the above grid system is converted into the following rectangular grid system.

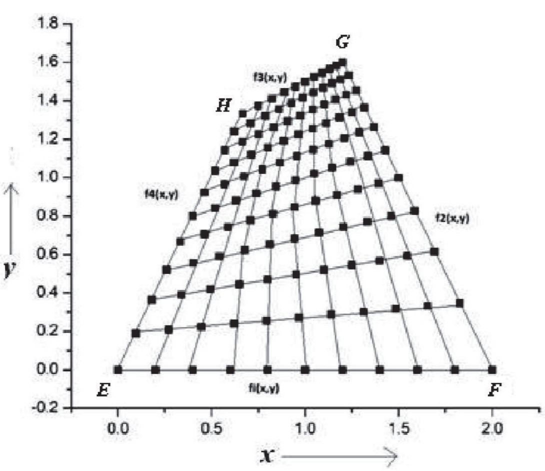

Fig.1(a): Structured irregular grid distribution of case study I

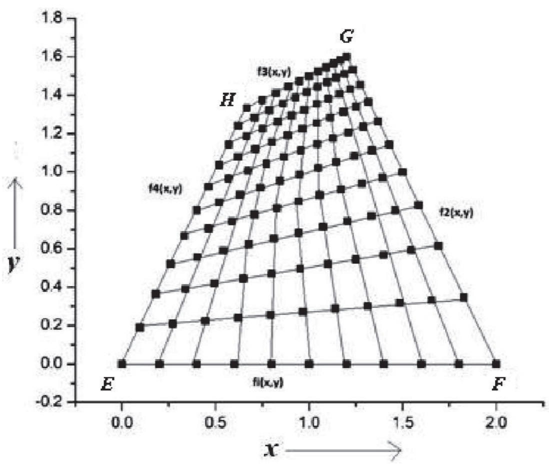

Fig.1(b): Structured regular grid distribution of case study I

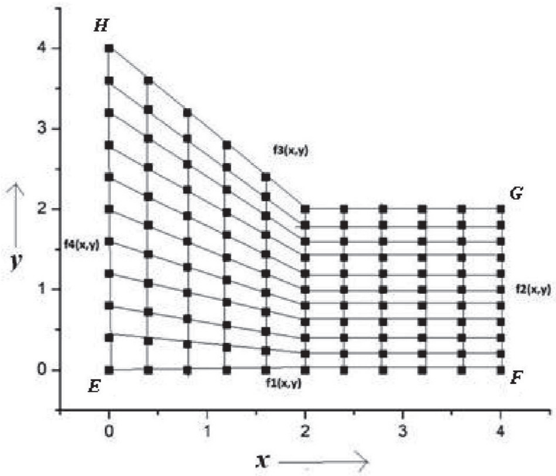

Fig.2(a): Structured irregular grid distribution of case study II

3.2 Case II

Let the irregular domain as shown in the Fig.2(a) is taken. Its faces EF, FG, GH, HE are represented by equations $f_1(x,y) = 0$, $f_2(x,y) = 0$, $f_3(x,y) = 0$ and $f_4(x,y) = 0$ respectively. The equations of the four faces are-

$$f_1(x,y) = y = 0 \tag{9}$$

$$f_2(x,y) = x\text{-}4 = 0 \tag{10}$$

$$\begin{aligned} f_3(x,y) &= x+y-4 = 0 \text{ for } 0 \le x \le 1 \\ &= y-2=0 \text{ for } 1 \le x \le 2 \end{aligned} \tag{11}$$

$$f_4(x,y) = x = 0 \tag{12}$$

By using the algebraic method described earlier,

$$\begin{aligned} p &= y/4\text{-}x \text{ for } 0 \le x \le 1 \\ &= y/2 \text{ for } 1 \le x \le 2 \end{aligned} \tag{13}$$

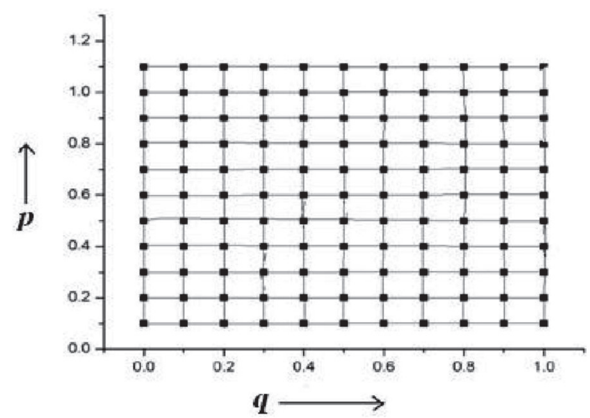

Fig.2(b): Structured regular grid distribution of case study II

$$q = x/4 \tag{14}$$

By inverse relationships,

$$x = 4q \tag{15}$$

$$\begin{aligned} y &= 4(q\text{-}2pq) \text{ for } 0 \le x \le 1 \\ &= 2q \text{ for } 1 \le x \le 2 \end{aligned} \tag{16}$$

By employing transfinite algorithm the above grid system is converted into the following rectangular grid system.

3.3 Case III

Let one consider an H type of domain. Let the irregular domain as shown in the Fig.3(a) is taken. Its faces EF, FG, GH, HE are represented by equations $f_1(x,y) = 0$, $f_2(x,y) = 0$, $f_3(x,y) = 0$ and $f_4(x,y) = 0$ respectively. The equations of the four faces are-

$$f_1(x,y) = y = 0 \text{ for } 0 \le x \le 2$$
$$= y-2 = 0 \text{ for } 3 \le x \le 7$$
$$= y = 0 \text{ for } 8 \le x \le 10 \quad (17)$$

$$f_2(x,y) = x-8 = 0 \quad (18)$$

$$f_3(x,y) = y-8 = 0 \text{ for } 0 \le x \le 2$$
$$= y-6 = 0 \text{ for } 3 \le x \le 7$$
$$= y-6 = 0 \text{ for } 8 \le x \le 10 \quad (19)$$

$$f_4(x,y) = x = 0 \quad (20)$$

By using the algebraic method described earlier,

$$p = x/8 \quad (21)$$

$$q = y/8 \text{ for } 0 \le x \le 2$$
$$= y-2/4 \text{ for } 3 \le x \le 7$$
$$= y/8 \text{ for } 8 \le x \le 10 \quad (22)$$

By inverse relationships,

$$x = 8p \quad (23)$$

$$y = 8q \text{ for } 0 \le x \le 2$$
$$= 4q+2 \text{ for } 3 \le x \le 7$$
$$= 8q \text{ for } 8 \le x \le 10 \quad (24)$$

By employing transfinite algorithm the grid system shown in Fig.3(a) is converted into the following rectangular grid system as shown in Fig.3(b).

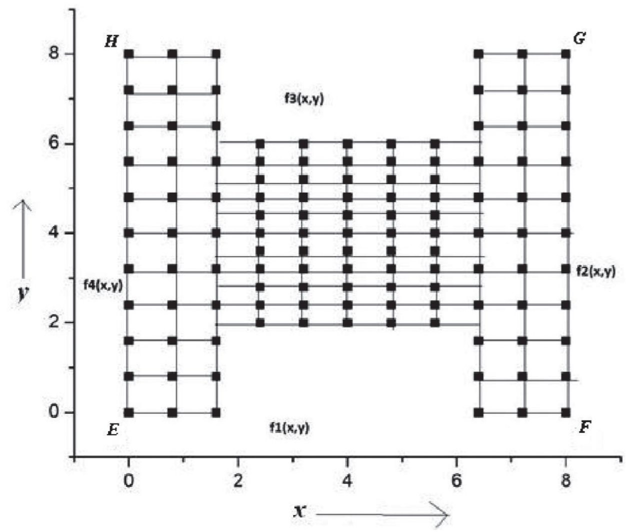

Fig.3(a): Structured irregular grid distribution of case study III

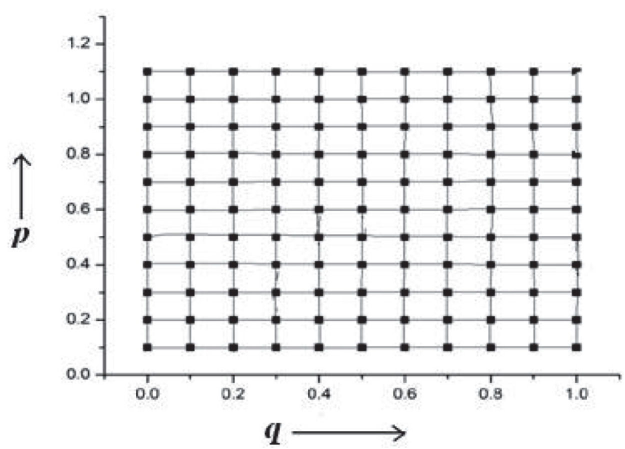

Fig.3(b): Structured regular grid distribution of case study III

4. DISCUSSION

Proper type of grid transformation depends upon many parameters. Codes based on these transformed grids must be robust, converging, requires less computation time. To satisfy the above code parameters, required computational domain must be simply structured, regular and rectangular in shape. Boundary conditions are also to be applied with ease for these kinds of transformed grids. Also, this transformation of grids decreases the solver timing to execute the final result.

The shapes of the domain discussed in Case I and in Case III are commonly found in structural foundation, different machine parts etc. To perform finite element analysis on this type of shape, the grids must be transformed into simple rectangular pattern to analyze the problem easily. The algorithm in this present work is very much appropriate for this transformation.

The shapes of the domain discussed in Case II are commonly found in pipe fitting, open channel flow etc. To perform computational fluid dynamics and heat transfer analyses on this kind of shape, the grids must be transformed into simple rectangular pattern to analyze the problem easily. The algorithm in this present work is also very much appropriate for this transformation.

5. CONCLUSION

Significant progress has been achieved in present days in the domain of algebraic grid transformation techniques. The most discussed gain is the regular unstructured grid formation approach. It is a very much flexible technique in different applications. Structured grid approaches are being applied due to their efficacy and sensitivity. The algorithm of grid transformation discussed in this present work is very much helpful in the computational field for its simple and easy to understand steps. Though there are several limitations of this algorithm. It is applied in 2 dimensional cases with rectangular shape grids. This algorithm can be modified in its most generalized form to fit in 3 dimensional cases with both rectangular and curved shapes of grids.

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